

# Reinforcement Learning

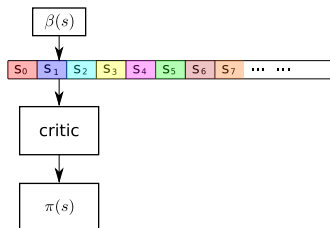
## 6. Replay buffer, Biases, Bias-Variance, Monte Carlo and Model-Based RL

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## Introducing a replay buffer



- ▶ Helps decorrelating the agent trajectory and samples fed to the critic
- ▶ Samples can be fed to the critic randomly or through various heuristics
- ▶ Introduces sample efficiency discussion

## Replay buffer and sample efficiency

- ▶ Important intuition: in the discrete deterministic case, one sample from each (state, action) pair in the buffer is enough for Q-LEARNING to converge
- ▶ Thus using a replay buffer can be very sample efficient
- ▶ In the stochastic case, samples in the replay buffer should reflect the distribution over next state
- ▶ This may require a large replay buffer (over  $1e^6$  samples)
- ▶ In the continuous case, the state (and action) spaces cannot be covered
- ▶ But off-policy deep RL algorithms using a replay buffer still benefit from the initial intuition

## Maximization in RL

- ▶ Two maximization steps:

- ▶ In action selection:

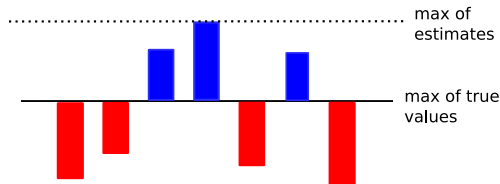
$$\pi(s) \sim \operatorname{argmax}_{a \in A} Q(s, a)$$

- might be stochastic or contain some exploration

- ▶ In Q-LEARNING, in the value update rule

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_{a \in A} Q(s_{t+1}, a) - Q(s_t, a_t)]$$

## Maximization bias

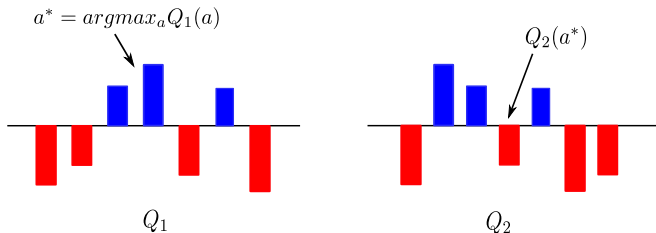


- ▶ In action selection, a maximum over estimated  $Q(s, a)$  is performed
- ▶ “In these algorithms, a maximum over estimated values is used implicitly as an estimate of the maximum value, which can lead to a significant positive bias.”
- ▶ Example: imagine all true  $Q(s, a)$  values are null



Sutton, R. S. & Barto, A. G. (2018) *Reinforcement Learning: An Introduction (Second edition)*. MIT Press

## Double Q-LEARNING

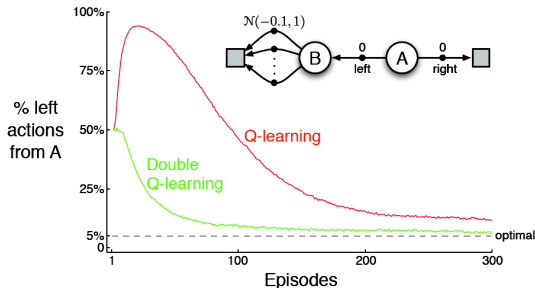


- ▶ Solution: using two Q-Tables, one for value estimation and one for action selection
- ▶  $a^* = \operatorname{argmax}_a Q_1(a)$
- ▶  $Q_2(a^*) = Q_2(\operatorname{argmax}_a Q_1(a))$  unbiased estimate of  $Q(a^*)$
- ▶  $a'^* = \operatorname{argmax}_a Q_2(a)$
- ▶  $Q_1(a'^*) = Q_1(\operatorname{argmax}_a Q_2(a))$  unbiased estimate of  $Q(a'^*)$
- ▶ Randomly select one of each at all steps



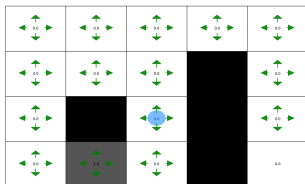
Van Hasselt, H. (2010) Double q-learning. *Advances in Neural Information Processing Systems*, pages 2613–2621

## Double Q-LEARNING: results



**Figure 6.5:** Comparison of Q-learning and Double Q-learning on a simple episodic MDP (shown inset). Q-learning initially learns to take the left action much more often than the right action, and always takes it significantly more often than the 5% minimum probability enforced by  $\epsilon$ -greedy action selection with  $\epsilon = 0.1$ . In contrast, Double Q-learning is essentially unaffected by maximization bias. These data are averaged over 10,000 runs. The initial action-value estimates were zero. Any ties in  $\epsilon$ -greedy action selection were broken randomly.

## Over-estimation bias propagation



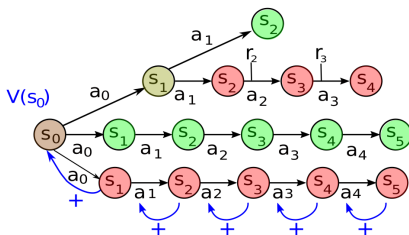
- ▶ Some initial bias cannot be prevented due to Q-Table initialization
- ▶ In Q-LEARNING, due to the max operator, the maximization bias propagates
- ▶ No propagation of under-estimation
- ▶ The same holds for DDPG without a max operator!



Fujimoto, S., van Hoof, H., & Meger, D. (2018) Addressing function approximation error in actor-critic methods. *arXiv preprint arXiv:1802.09477*



## Monte Carlo (MC) methods



- ▶ Much used in games (Go...) to evaluate a state
- ▶ It uses the average estimation method  $E_{k+1}(s) = E_k(s) + \alpha[r_{k+1} - E_k(s)]$
- ▶ Generate a lot of trajectories:  $s_0, s_1, \dots, s_N$  with observed rewards  $r_0, r_1, \dots, r_N$
- ▶ Update state values  $V(s_k)$ ,  $k = 0, \dots, N - 1$  with:

$$V(s_k) \leftarrow V(s_k) + \alpha(s_k)(r_k + r_{k+1} + \dots + r_N - V(s_k))$$

## TD vs MC

- ▶ Temporal Difference (TD) methods combine the properties of DP methods and Monte Carlo methods:
- ▶ In Monte Carlo,  $T$  and  $r$  are **unknown**, but the value update is **global** along **full trajectories**
- ▶ In DP,  $T$  and  $r$  are **known**, but the value update is **local**
- ▶ TD: as in DP,  $V(s_t)$  is updated **locally** given an estimate of  $V(s_{t+1})$  and  $T$  and  $r$  are **unknown**
- ▶ Note: Monte Carlo can be reformulated incrementally using the temporal difference  $\delta_k$  update

## Eligibility traces

- ▶ Goal: improve over Q-LEARNING
- ▶ Naive approach: store all  $(s, a)$  pair and back-propagate values
- ▶ Limited to finite horizon trajectories
- ▶ Speed/memory trade-off
- ▶  $TD(\lambda)$ ,  $SARSA(\lambda)$  and  $Q(\lambda)$ : more sophisticated approach to deal with infinite horizon trajectories
- ▶ A variable  $e(s)$  is decayed with a factor  $\lambda$  after  $s$  was visited and reinitialized each time  $s$  is visited again
- ▶  $TD(\lambda)$ :  $V(s) \leftarrow V(s) + \alpha \delta e(s)$ , (similar for  $SARSA(\lambda)$  and  $Q(\lambda)$ ),
- ▶ If  $\lambda = 0$ ,  $e(s)$  goes to 0 immediately, thus we get  $TD(0)$ ,  $SARSA$  or Q-LEARNING
- ▶  $TD(1) =$  Monte Carlo...
- ▶ Eligibility traces can be seen as a combination of N-step returns for all  $N$



Schulman, J., Moritz, P., Levine, S., Jordan, M., & Abbeel, P. (2015b) High-dimensional continuous control using Generalized Advantage Estimation. *arXiv preprint arXiv:1506.02438*



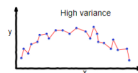
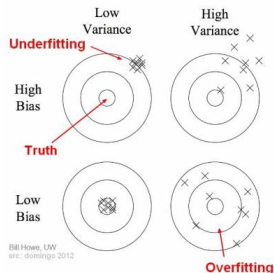
## Bias versus variance

- ▶ If you compute an expectation over infinitely many samples, you get the same expectation each time
- ▶ But if you compute it over a finite set of samples, you get a different expectation each time
- ▶ This is known as **variance**
- ▶ Given a large variance, you need many samples to get an accurate estimate of the mean
- ▶ If you update an expectation based on a previous (wrong) expectation estimate, the expectation estimate you get provided infinitely many samples is wrong
- ▶ This is known as **bias**

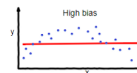


Geman, S., Bienenstock, E., & Doursat, R. (1992) Neural networks and the bias/variance dilemma. *Neural computation*, 4(1):1–58

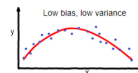
# Bias, variance, overfitting and underfitting



overfitting



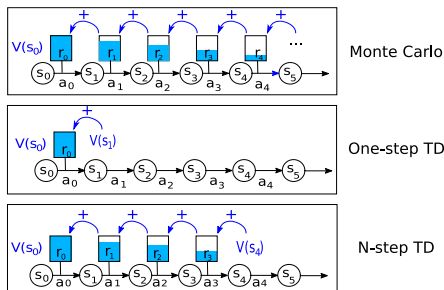
underfitting



Good balance

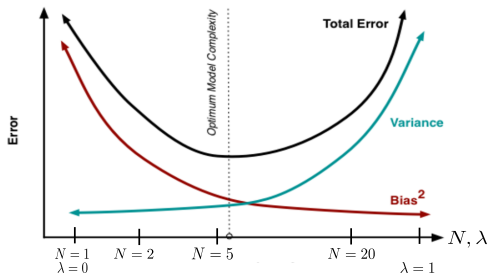
- ▶ With high bias, the risk is underfitting
- ▶ With high variance, the risk is overfitting
- ▶ You need low bias and low variance

# Monte Carlo, One-step TD and N-step return



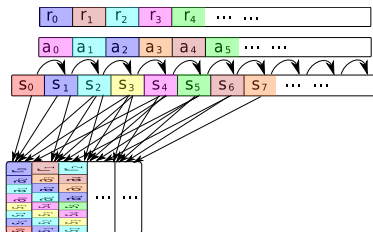
- ▶ One-step TD suffers from bias
- ▶ MC suffers from variance due to exploration (+ stochastic trajectories)
- ▶ MC is on-policy → less sample efficient
- ▶ N-step TD: tuning  $N$  to control the bias-variance compromise

# Bias-variance compromise



- ▶ Total error = bias<sup>2</sup> + variance + irreducible error
- ▶ A more complex model (e.g. bigger network) generally has more variance, but less bias
- ▶ Tuning  $N$  in the  $N$ -step return or  $\lambda$  in an eligibility trace method helps finding the right compromise.

## The N-step return in practice



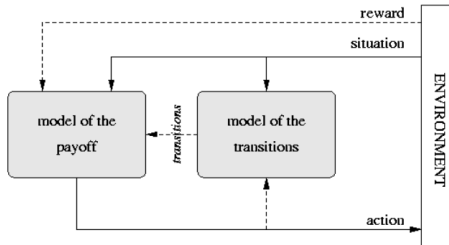
- ▶ How do we store into the replay buffer?
- ▶ N-step Q-LEARNING is more efficient than Q-LEARNING
- ▶ Various implementations are possible



Sharma, S., Ramesh, S., Ravindran, B., et al. (2017) Learning to mix N-step returns: Generalizing  $\lambda$ -returns for deep reinforcement learning. *arXiv preprint arXiv:1705.07445*



# Model-based Reinforcement Learning

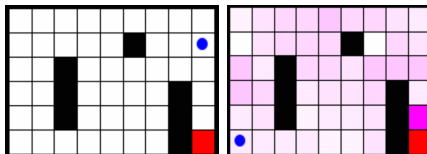


- ▶ General idea: planning with a learnt model of  $T$  and  $r$  is performing back-ups “in the agent’s head” ([Sutton, 1990, Sutton, 1991])
- ▶ Learning  $T$  and  $r$  is an incremental **self-supervised** learning problem
- ▶ Several approaches:
  - ▶ Draw random transition in the model and apply TD back-ups
  - ▶ DYNA-PI, DYNA-Q, DYNA-AC
  - ▶ Better propagation: Prioritized Sweeping



Moore, A. W. & Atkeson, C. (1993). Prioritized sweeping: Reinforcement learning with less data and less real time. *Machine Learning*, 13:103–130.

## Dyna architecture and generalization



- ▶ Thanks to the model of transitions, DYNA can propagate values more often
- ▶ Problem: in the stochastic case, the model of transitions is in  $\text{card}(S) \times \text{card}(S) \times \text{card}(A)$
- ▶ Usefulness of **compact** models
- ▶ MACS: DYNA with generalisation (Learning Classifier Systems)
- ▶ SPITL: DYNA with generalisation (Factored MDPs)



Gérard, P., Meyer, J.-A., & Sigaud, O. (2005) Combining latent learning with dynamic programming in MACS. *European Journal of Operational Research*, 160:614–637.



Degris, T., Sigaud, O., & Wuillemin, P.-H. (2006) Learning the Structure of Factored Markov Decision Processes in Reinforcement Learning Problems. *Proceedings of the 23rd International Conference on Machine Learning (ICML'2006)*, pages 257–264

## Corresponding labs

- ▶ See [https://github.com/osigaud/rl\\_labs\\_notebooks](https://github.com/osigaud/rl_labs_notebooks)
- ▶ One notebook about N-step return
- ▶ One notebook about model-based RL, based on RTDP

Any question?



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Integrating architectures for learning, planning, and reacting based on approximating dynamic programming.

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*SIGART Bulletin*, 2:160–163.



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