AMPyC

Chapter 4: Nonlinear robust MPC - Part II

30. May 2025

Authors: Prof. Melanie Zeilinger
Dr. Johannes Köhler
Dr. Andrea Carron
Dr. Lukas Hewing

ETH Zurich

Robust MPC

- Previous chapter: “Nonlinear robust tube-based MPC”
  - Derive a robust positive invariant (RPI) set using incremental stability
  - Tube-based MPC guarantees recursive feasibility, robust constraint satisfaction, and convergence to a small neighborhood given a bound $\mathcal{W}$ on the disturbance.

- What is missing:
  - Robust MPC can be conservative.
    (Large disturbances w may result in a small feasible set due to the constraint tightening.)

- Goal:
  - Reduce conservatism by incorperating more knowledge about model mismatch.

Contents - 4: Robust tube-MPC

- Uncertainty characterization in robust MPC
    - Understand why constant bounds on disturbance magnitude can be conservative
    - State & input dependent disturbance bounds
    - Design robust MPC by avoiding uncertain areas
- Incorporate state & input dependent disturbance bound in MPC
    - Understand concept of predicting homothetic tubes
    - Incorporate state & input dependent disturbance bound in tube-MPC
    - Prove closed loop properties

Outline

1. Tube-MPC: Avoiding areas with large model-mismatch

2. Homothetic tube-MPC using state & input dependent disturbance bounds

3. Alternative parameterizations in the literature

Outline

1. Tube-MPC: Avoiding areas with large model-mismatch

    Uncertainty characterization in robust MPC

    State & input dependent disturbance bounds

    Tube-MPC with state & input dependent disturbances

Uncertainty characterization in robust MPC

- $x(k+1) = f(x(k), u(k)) + w(k)$ with known nonlinear model f and disturbances w.
- Simple disturbance bound: $w(k) \in \mathcal{W}$

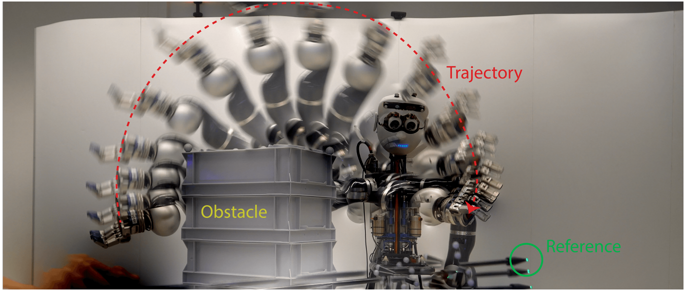
Sources of model mismatch encountered in applications (e.g., autonomous driving car)

- Additive unpredictable disturbances (e.g., wind)
- Measurement noise (e.g., quantization error in sensor)
- Parametric uncertainty (e.g., friction coefficient not exactly estimated)
- Unmodelled/simplified nonlinearities (e.g., simplified linear friction model)
- Dynamic uncertainty (e.g., lower order model ignoring actuator dynamics)
- ...

$\Rightarrow$ Bounding all model-mismatch with a constant $\bar{w}$ is simple, but conservative.

Uncertainty characterization in robust MPC

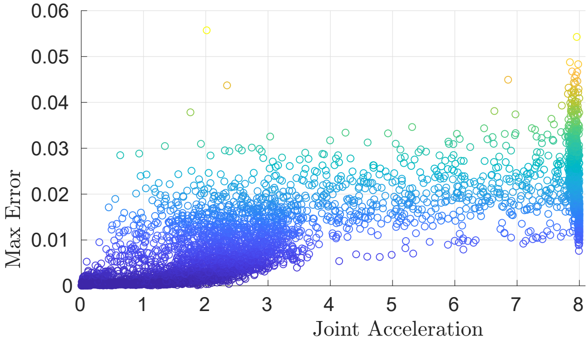
Example – robotic manipulator [1]



- Robotic manipulator
- Sources of model mismatch: noisy measurements; estimated inertia matrix; unmodelled actuator dynamics; . . .

Uncertainty characterization in robust MPC

Example – robotic manipulator [1]



- Disturbance magnitude highly correlated to acceleration
- Constant disturbance bound might be conservative
- Incorporate the knowledge in the design

How would you use this data in a robust MPC design?

Outline

1. Tube-MPC: Avoiding areas with large model-mismatch

   Uncertainty characterization in robust MPC

   State & input dependent disturbance bounds

   Tube-MPC with state & input dependent disturbances

Uncertainty characterization in robust MPC

- State and input dependent (static[1]) disturbance bound:
  $w(k) \in \mathcal{W}(x(k), u(k))$.
- Constant bound $w(k) \in \overline{\mathcal{W}} = \{w \in \mathcal{W}(x, u) \mid \exists (x, u) \in \mathcal{Z}\}$ is more conservative
- How to obtain such an disturbance bound?
  $\Rightarrow$ Will be discussed more in second half of this course
  (e.g., bayesian linear regression, set-membership estimation).
- How can we use this knowledge?

[1] Dynamic uncertainty or estimation error requires modification (cf. [2, 3], [4]).

RPI sets with state & input dependent disturbances [5][2]

- incremental ISS Lyapunov function:
  - $\|x-z\|_{\underline{M}} \leq V_\delta(x,z) \leq \|x-z\|_{\overline{M}}, \quad \underline{M}, \overline{M} \succ 0.$
  - $V_\delta(f(x,\kappa(x,z,v)) + w, f(z,v)) \leq \rho V_\delta(x,z) + \|w\|_{\overline{M}}, \quad \rho \in [0,1).$

- If $\|w(k)\|_{\overline{M}} \leq \overline{w}$, then $\Omega = \left\{(x,z) | V_\delta(x,z) \leq \delta = \frac{\overline{w}}{1-\rho}\right\}$ is robust positive invariant (RPI).

- **Idea:** Impose suitable constraint on nominal trajectory, such that $\|w(k)\|_{\overline{M}} \leq \overline{w}$ with specified disturbance level $\overline{w} > 0$,
  i.e., avoid areas with too large model uncertainty
  $\Rightarrow$ RPI set Ω designed offline based on smaller disturbance bound $\overline{w}$.

[2] Similar ideas in [6] for dynamic uncertainty.

RPI sets with state & input dependent disturbances [5]

- Disturbance bound:
  - $\|w(k)\|_{\overline{M}} \leq \tilde{w}(x(k), u(k))$ with $\tilde{w}$ Lipschitz continuous.
  - $\tilde{w}(x, \kappa(x, z, v)) \leq \tilde{w}(z, v) + L_{\mathrm{w}} V_\delta(x, z)$ with $L_{\mathrm{w}} \geq 0$.

$\Rightarrow$ We can ensure $\|w(k)\|_{\overline{M}} \leq \overline{w}$ using a (tightened) nominal constraint: $\tilde{w}(z, v) + L_{\mathrm{w}}\delta \leq \overline{w}$.

- Special case: $w = G(x)\theta$, with some uncertain paramteter $|\theta| \leq 1$.
  - Simple upper bound given by: $\tilde{w}(x, u) = \|\overline{M}^{1/2} G(x)\|$.
  - Continuity constant: $L_{\mathrm{w}} = \max_{x \in \mathcal{X}} \|\overline{M}^{1/2} \left[\frac{\partial G}{\partial x}\right]\Big|_x \underline{M}^{-1/2}\|$.

- Practical example:
  - $x = [p; v]$ with position p, velocity v.
  - Linear friction with uncertain coefficient: $w = v \cdot \theta$.
  - Maximal disturbance proportional to velocity $|v|$.

  $\Rightarrow$ By limiting the maximal velocity we can get a bound on the maximal model mismatch.

Outline

1. Tube-MPC: Avoiding areas with large model-mismatch

    Uncertainty characterization in robust MPC

    State & input dependent disturbance bounds

    Tube-MPC with state & input dependent disturbances

Tube-MPC with state & input dependent disturbances [5]

$$
\begin{aligned}
V_N^\star(x) = \min_{V,\, z_0} \quad & \sum_{i=0}^{N-1} l(z_i, v_i) + l_{\mathrm{f}}(z_N) \\
\text{s.t.} \quad & z_{i+1} = f(z_i, v_i), \quad i \in [0, N-1], \\
& g_j(z_i, v_i) + c_j \delta \leq 0, \quad i \in [0, N-1],\ j \in [1, p], \\
& \tilde{w}(z_i, v_i) + L_{\mathrm{w}} \delta \leq \overline{w}, \quad i \in [0, N-1], \\
& z_N \in \mathcal{X}_{\mathrm{f}}, \\
& V_\delta(x, z_0) \leq \delta = \frac{\overline{w}}{1-\rho}
\end{aligned}
$$

- Constraint tightening and initial state constraint proportional to δ.
- maximal disturbance bound $\overline{w} > 0$ enforced using bound on $\tilde{w}(z, v)$.

Overall algorithm
Tube-MPC using state & input dependent disturbance bound

— Offline —

1. Design a stabilizing feedback κ and an incremental Lyapunov function V_δ.
2. Obtain state & input dependent disturbance bound $\tilde{w}(x,u)$ and set a maximal bound $\overline{w} > 0$.
3. Compute constants $\delta, c_j, L_\mathrm{w} > 0$.
4. Design terminal cost V_f and terminal set $\mathcal{X}_\mathrm{f}$ for nominal system with tightened constraints[3].

— Online—

1. Measure/estimate state x.
2. Solve tube-MPC problem.
3. Apply $u = \kappa(x, z_0^\star(x), v_0^\star(x))$.

[3]Including the tightened constraint on $\tilde{w}(z,v)$, i.e., $\tilde{w}(z,\pi_\mathrm{f}(z)) + L_\mathrm{w}\delta \leq \overline{w}$ for all $z \in \mathcal{X}_\mathrm{f}$.

Tube-MPC - Theoretical analysis

Theorem: Tube-MPC with state & input dependent disturbance bounds

Let assumptions hold and assume that the tube-MPC problem is feasible at $k = 0$ with initial condition $x(0)$. Then, the tube-MPC problem is recursively feasible and the constraints are satisfied for all $k \geq 0$, for the closed-loop system $x(k+1) = f(x(k), \mu_{\mathrm{tube}}(x(k)) + w(k)$. Furthermore, the closed loop converges to the RPI set $\{x | V_\delta(x, 0) \leq \delta\}$.

Proof: Let $(\{(v_0^\star, \ldots, v_{N-1}^\star\}, \{z_0^\star, \ldots, z_N^\star\})$ be the optimal solution at time k. Feasibility ensures

$$
\begin{aligned}
\|w(k)\|_{\overline{M}} \leq & \tilde{w}(x(k), u(k)) \leq \tilde{w}(z_0^\star, v_0^\star) + L_{\mathrm{w}} V_\delta(x(k), z_0^\star) \\
\leq & \tilde{w}(z_0^\star, v_0^\star) + L_{\mathrm{w}} \delta \leq \overline{w}.
\end{aligned}
$$

$\Rightarrow$ At the next time step, we have $V_\delta(x(k+1), z_1^\star) \leq \delta$ due to the RPI design.
The remaining properties follow analogous to tube-MPC from the last lecture using the same candidate solution. $\square$

Tube-MPC using state & input dependent disturbance bound - Discussion

Pro:

- Incorporate state & input dependent disturbance bound in the design
- Avoid areas with large disturbances ($\tilde{w}(x,u) \leq \overline{w}$)
- Simple design: Fix disturbance bound using tuning variable $\overline{w} > 0$.
- Same theoretical properties as "standard" tube-MPC (robust constraint satisfaction and robust stability/performance), but avoid conservatism of globally valid disturbance bound.

Limitation:

- Maximal disturbance bound $\overline{w}$ needs to be chosen offline.
  $\Rightarrow$ Can be unnecessarily cautious
  (Avoid areas with large uncertainty, even if safe operation possible)

Outline

1. Tube-MPC: Avoiding areas with large model-mismatch
2. Homothetic tube-MPC using state & input dependent disturbance bounds
3. Alternative parameterizations in the literature

Homothetic tubes

- Directly utilize disturbance bound $\tilde{w}(x, u)$ to determine if a nominal planned trajectory is "safe".

- Problem: Tube-MPC uses a constant (rigid) tube $(x, z) \in \Omega$,
  but disturbance bound $\tilde{w}(x, u)$ changes along the prediction horizon.

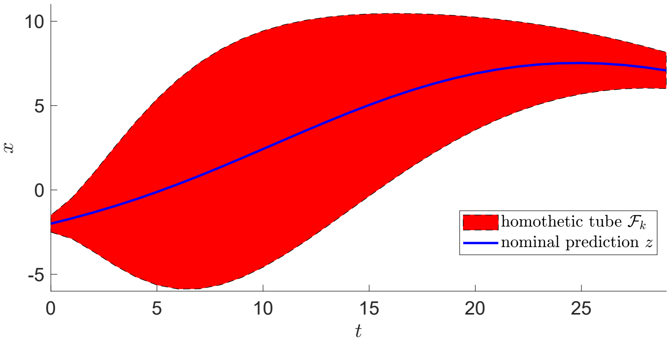
$\Rightarrow$ Use more flexible "homothetic" tube formulation [7, 8].

Homothetic tubes

- Tube parametrization using a scaling $\delta_k > 0$:

$$\mathcal{F}_k = \{x | V_\delta(x, z_k) \leq \delta_k\}.$$

- Predict tube-size δ_k using the disturbance bound $\tilde{w}(x, u)$.



Homothetic tubes

Tube dynamics [9]

Consider the tube propagation: $\delta_{k+1} = \tilde{\rho}\delta_k + \tilde{w}(z_k, v_k)$, $V_\delta(x, z_0) \leq \delta_0$.
Then, $V_\delta(x(k), z_k) \leq \delta_k$ holds $k \geq 0$.

Derivation: $x^+ = f(x, u) + w$, $u = \kappa(x, z, v)$, $z^+ = f(z, v)$ with disturbance $\|w\|_{\overline{M}} \leq \tilde{w}(x, u)$:

$$\begin{aligned} V_\delta(x^+, z^+) \leq & \rho V_\delta(x, z) + \|w\|_{\overline{M}} \\ \leq & \rho V_\delta(x, z) + \tilde{w}(x, u) \\ \leq & \rho V_\delta(x, z) + \tilde{w}(z, v) + L_\mathrm{w} V_\delta(x, z) \\ = & \underbrace{(\rho + L_\mathrm{w})}_{=\tilde{\rho}} V_\delta(x, z) + \tilde{w}(z, v). \end{aligned}$$

Contraction rate for uncertain system $\tilde{\rho} < 1$ for L_w sufficiently small.

Induction: $V_\delta(x(k+1), z_{k+1}) \leq \tilde{\rho} V_\delta(x(k), z_k) + \tilde{w}(z_k, v_k) \leq \tilde{\rho}\delta_k + \tilde{w}(z_k, v_k) = \delta_{k+1}$.

Homothetic tube-MPC using state & input dependent disturbance bounds [9, 10]

$$
\begin{aligned}
V_N^\star(x) = \min_{V, z_0} \quad & \sum_{i=0}^{N-1} l(z_i, v_i) + l_\mathrm{f}(z_N) \\
\text{s.t.} \quad & z_{i+1} = f(z_i, v_i), \quad i \in [0, N-1], \\
& \delta_{i+1} = \tilde{\rho}\delta_i + \tilde{w}(z_i, v_i), \quad i \in [0, N-1], \\
& g_j(z_i, v_i) + c_j \delta_i \leq 0, \quad i \in [0, N-1],\ j \in [1, p], \\
& (z_N, \delta_N) \in \mathcal{X}_\mathrm{f} \times [0, \bar{\delta}], \\
& \delta_0 = V_\delta(x, z_0)
\end{aligned}
$$

- Tube size δ_i deterministically predicted along horizon based on initial value δ_0 and nominal disturbance bound $\tilde{w}(z, v)$.
- Constraint tightening and terminal set constraint depend on tube size δ_i.

Offline design

- incremental ISS with Lipschitz continuous feedback κ:
  - $\|x - z\|_{\underline{M}} \leq V_\delta(x, z) \leq \|x - z\|_{\overline{M}}$
  - $V_\delta(f(x, \kappa(x, z, v)) + w, f(z, v)) \leq \rho V_\delta(x, z) + \|w\|_{\overline{M}}$
- Lipschitz continuous disturbance bound $\tilde{w}$:
  - $\|w(k)\|_{\overline{M}} \leq \tilde{w}(x(k), u(k))$.
- Terminal set: For any $(z, \delta) \in \mathcal{X}_\mathrm{f} \times [0, \overline{\delta}]$, we have
  - Positive invariance: $f(z, \pi_\mathrm{f}(z)) \in \mathcal{X}_\mathrm{f}$, $\tilde{\rho}\delta + \tilde{w}(z, \pi_\mathrm{f}(z)) \leq \overline{\delta}$. [4]
  - Constraint satisfaction: $g_j(z, \pi_\mathrm{f}(z)) + c_j\delta \leq 0$, $j \in [1, p]$.
  - Local Lyapunov function: $l_\mathrm{f}(f(z, \pi_\mathrm{f}(z))) - l_\mathrm{f}(z) \leq -l(z, \pi_\mathrm{f}(z))$.
- Positive definite and bounded cost:
  - $l(x, u) \geq \alpha_l(\|x\|)$, $\forall (x, u) \in \mathcal{X} \times \mathcal{U}$, $\alpha_l \in \mathcal{K}_\infty$,
  - $0 \in \mathrm{int}(\mathcal{X}_\mathrm{f})$ and $l_\mathrm{f}(x) \leq \alpha_\mathrm{f}(\|x\|)$, $\forall x \in \mathcal{X}_\mathrm{f}$, $\alpha_\mathrm{f} \in \mathcal{K}_\infty$.

[4] Given $\tilde{\rho} < 1$, this requires $\overline{\delta} \geq \max_{z \in \mathcal{X}_\mathrm{f}} \tilde{w}(z, \pi_\mathrm{f}(z))/(1 - \tilde{\rho})$.

Homothetic tube-MPC- Theoretical analysis

Theorem: Homothetic tube-MPC with state & input dependent disturbance bounds

Let assumptions hold and assume that the homothetic tube-MPC problem is feasible at $k = 0$ with initial condition $x(0)$. Then, the homothetic tube-MPC problem is recursively feasible and the constraints are satisfied for all $k \geq 0$, for the closed-loop system $x(k+1) = f(x(k), \mu_{\mathrm{tube}}(x(k)) + w(k)$. Furthermore, the closed loop converges to the RPI set $\{x | V_\delta(x, 0) \leq \overline{\delta}\}$.

Homothetic tube-MPC- Theoretical analysis

Proof: Let $(\{v_0^\star, \dots, v_{N-1}^\star\}, \{z_0^\star, \dots, z_N^\star\}, \{\delta_0^\star, \dots, \delta_N^\star\})$ be the optimal solution at time k. Consider the candidate the sequence

$$z_0 = z_1^\star, \quad V = \{v_1^\star, \dots, v_{N-1}^\star, \pi_\mathrm{f}(z_N^\star)\},$$

with z_i, δ_i according to equality constraints (dynamics). At time $k+1$, we have

$$\begin{aligned}
\delta_0 =& V_\delta(x(k+1), z_1^\star) = V_\delta(f(x(k), \kappa(x(k), v_0^\star, z_0^\star) + w(k)), f(z_0^\star, v_0^\star)) \\
\leq& \rho V_\delta(x(k), z_0^\star) + \|w(k)\|_{\overline{M}} \leq \rho V_\delta(x(k), z_0^\star) + \tilde{w}(x(k), u(k)) \\
\leq& (\rho + L_\mathrm{w}) V_\delta(x(k), z_0^\star) + \tilde{w}(z_0^\star, v_0^\star) = \tilde{\rho}\delta_0^\star + \tilde{w}(z_0^\star, v_0^\star) = \delta_1^\star.
\end{aligned}$$

Furthermore, the predicted scaling δ_i satisfies for any $i \in [0, N-2]$

$$\delta_{i+1} - \delta_{i+2}^\star = \tilde{\rho}(\delta_i - \delta_{i+1}^\star) = \tilde{\rho}^{i+1}(\delta_0 - \delta_1^\star) \leq 0,$$

i.e., we have $\delta_i \leq \delta_{i+1}^\star$, $i \in [0, N-1]$. $\Rightarrow$ New tube is contained in previously predicted tube.

Homothetic tube-MPC- Theoretical analysis

Proof - cont. This candidate satisfies the tightened state and input constraints using

$$g_j(z_i, v_i) + c_j\delta_i \leq g_j(z^\star_{i+1}, v^\star_{i+1}) + c_j\delta^\star_{i+1} \leq 0,$$

with $v^\star_N = \pi_{\mathrm{f}}(z^\star_N)$ and satisfaction for $i = N-1$ uses the conditions on the terminal set. Satisfaction of the terminal set constraint follows using the positive invariance condition of the terminal set with $\delta_{N-1} \leq \delta^\star_N$.

$\Rightarrow$ Candidate satisfies all constraints. Problem is recursively feasible.

State and input constraint satisfaction of the true state and input follows by the construction of the constraint tightening.

Analogous to the proof in the last lecture, we have $\lim_{k\to\infty} \|z^\star_0(k)\| = 0$ and $\lim_{k\to\infty} \|v^\star_0(k)\| = 0$.

Using $\delta^\star_0(k+1) \leq \delta^\star_1(k) = \tilde{\rho}\delta^\star_0(k) + \tilde{w}(z^\star_0(k), v^\star_0(k))$, we have $\limsup_{k\to\infty} \delta^\star_0(k) \leq \dfrac{\tilde{w}(0,0)}{1-\tilde{\rho}} \leq \overline{\delta}$.

The initial state constraint ensures that the state converges to the RPI set with $V_\delta(x, 0) \leq \overline{\delta}$. $\square$

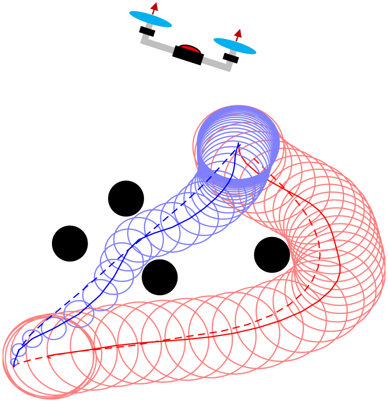
Homothetic tube-MPC - Discussion

- Directly use state & input dependent disturbance bound in the MPC design using a homothetic tube
- More flexible, less conservative, larger set of feasible states compared to previous tube-MPC.
- Same theoretical properties as "standard" tube-MPC
  (robust constraint satisfaction and robust stability/performance).
- Implicitly avoid areas with large disturbances ($\tilde{w}$) using tube-dynamics and back-off.
- Offline design complexity similar to tube-MPC.
- Online computation complexity compared to nominal MPC:
  Additionally requires evaluation of functions $V_\delta(x, z)$, $\tilde{w}(z, v)$ to predict tube size δ_i

Homothetic tube-MPC - Example

Nonlinear quadrotor $x \in \mathbb{R}^6$: wind disturbances, uncertain model constants (mass)

- V_δ computed using SOS [11] (quadratic functions not applicable)
- Model-mismatch depends on thrust and angle
- Compare rigid tube to homothetic tube
- Homothetic tube more flexible
- Complexity increased due to $\tilde{w}(z, v)$ nonlinear



Plots taken from [11, 10]

Outline

1. Tube-MPC: Avoiding areas with large model-mismatch

2. Homothetic tube-MPC using state & input dependent disturbance bounds

3. Alternative parameterizations in the literature

Literature overview - robust MPC for nonlinear system

Constraint-tightening MPC [12, 9]:

Special case of homothetic tube: fix initial state $z_0 = x$, $\delta_0 = 0$.
Comparable to the linear constraint-tightening MPC in the last lecture.

Advantages

- V_δ does not directly appear in the MPC formulation.

$\Rightarrow$ Remove possibly complex initial state constraint.

$\Rightarrow$ Can be applied by estimating constants ρ instead of designing κ, V_δ [13].

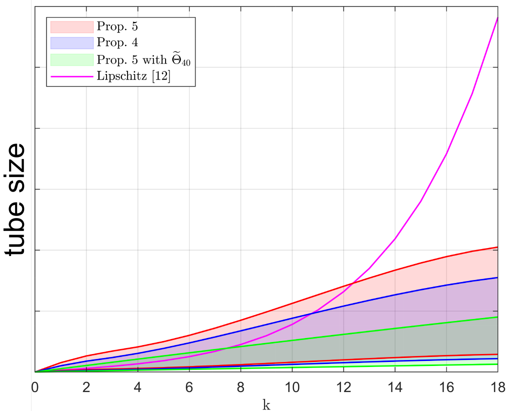
Drawbacks

- More restrictive initial state constraint $\Rightarrow$ smaller feasible set.
- Stronger assumptions needed for the terminal set.
- Weaker stability properties (ISS).

Literature overview - robust MPC for nonlinear system

Propagation using Lipschitz constant [14, 15]:

- Special case of constraint tightening with $V_\delta(x,z) = \|x - z\|$, $\kappa(x,z,v) = v$
- Easy to implement
- Contraction rate ρ replaced by Lipschitz constant L of nominal dynamics $f(x,u)$.
- Conservative for larger horizons ($L^N \gg 1$)

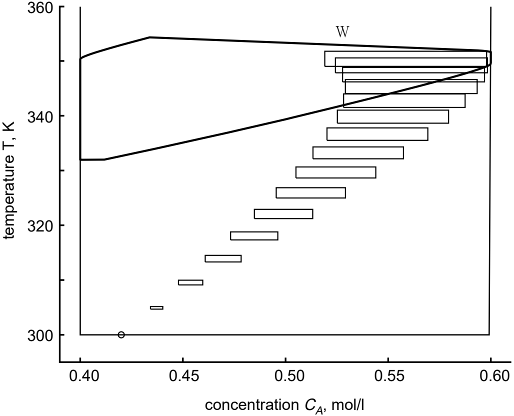


Plot taken from [16]

Literature overview - robust MPC for nonlinear system

Interval arithemtic [17]:

- Interval prediction
  $\mathcal{F}_k = \{x|\ x_j \in [\underline{x}_j(k), \overline{x}_j(k)]\}$.
- Easy to implement using interval arithmetic
- Interval-sets in general not invariant
  $\Rightarrow$ Can get very large/conservative

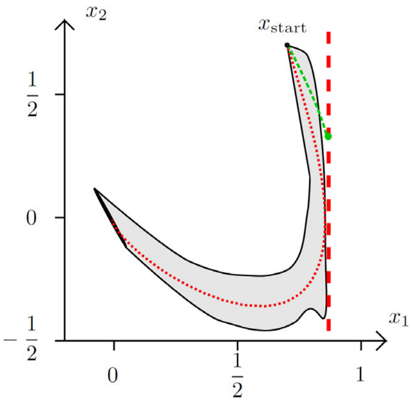


Plot taken from [17]

Literature overview - robust MPC for nonlinear system

Predict parametrized ellipsoidal tubes [18]:

- $\mathcal{F}_k = \{x | \ \|x - z_k\|^2_{P_k} \leq 1\}$ with online optimized matrices $P_k \in \mathbb{R}^{n \times n}$
- More degrees of freedom
- No system specific offline design required
- Larger computational complexity: $n^2/2 \cdot N$ decision variables (tailored numerical solutions exist [19])



Plot taken from [20]

Literature overview - robust MPC for nonlinear system

- Presented parametrization based on incremental Lyapunov function V_δ and scaling $\delta_i > 0$.
- Many other parametrization exist in the literature.
- Main tools (stabilizing feedback, over-approximation of the reachable set) are similar; but different parametrizations of the tubes and feedbacks[5].

$\Rightarrow$ Can trade-off computational complexity and conservatism by using simpler methods or more general/flexible parametrizations (cf., e.g., comparisons in [23, 24]).

[5] Some recent approaches do not require an offline choice of feedback κ, but optimize it online, thus reducing conservatism [20, 21, 22].

Robust MPC for nonlinear uncertain systems - summary

Idea:

- Compensate for noise in nominal prediction using a stabilizing feedback κ
- Method: tube based on incremental stability

Benefits:

- Tuneable trade-off robustness against performance using disturbance bound $\mathcal{W}$
- Guaranteed stability and safety properties
- Feasible set is invariant - we know exactly when the controller will work

Cons/Limitations:

- Often conservative $\Rightarrow$ State and input dependent disturbance bounds
- Offline design challenging for high-dimensional nonlinear systems
- How to obtain accurate deterministic disturbance bound $\mathcal{W}$?

Literature

- Wabersich, K. P., & Zeilinger, M. N. (2021). Nonlinear learning-based model predictive control supporting state and input dependent model uncertainty estimates. International Journal of Robust and Nonlinear Control.
- Köhler, J., Soloperto, R., Müller, M. A., & Allgöwer, F. (2020). A computationally efficient robust model predictive control framework for uncertain nonlinear systems. IEEE Transactions on Automatic Control.
- Sasfi A., Zeilinger, M. N., & Köhler, J. (2023). Robust adaptive MPC using control contraction metrics. Automatica.

References and further reading I

[1] Julian Nubert, Johannes Köhler, Vincent Berenz, Frank Allgöwer, and Sebastian Trimpe. Safe and fast tracking on a robot manipulator: Robust MPC and neural network control. **IEEE Robotics and Automation Letters**, 2020.

[2] Lukas Schwenkel, Johannes Köhler, Matthias A Müller, and Frank Allgöwer. Model predictive control for linear uncertain systems using integral quadratic constraints. **IEEE Trans. Automat. Control**, 2022.

[3] John Irvin Alora, Luis A. Pabon, Johannes Köhler, Mattia Cenedese, Edward Schmerling, Melanie N. Zeilinger, George Haller, and Marco Pavone. Robust nonlinear reduced-order model predictive control. In **Proc. 62nd IEEE Conf. Decision and Control (CDC)**, 2023.

[4] Johannes Köhler, Matthias A Müller, and Frank Allgöwer. Robust output feedback model predictive control using online estimation bounds. **arXiv**, 2021.

[5] Kim P Wabersich and Melanie N Zeilinger. Nonlinear learning-based model predictive control supporting state and input dependent model uncertainty estimates. **Int. J. Robust and Nonlinear Control**, 2021.

[6] Paola Falugi and David Q Mayne. Getting robustness against unstructured uncertainty: a tube-based MPC approach. **IEEE Trans. Automat. Control**, 2014.

References and further reading II

[7] Saša V Raković, Basil Kouvaritakis, Rolf Findeisen, and Mark Cannon. Homothetic tube model predictive control. **Automatica**, 2012.

[8] Saša V Raković, Li Dai, and Yuanqing Xia. Homothetic tube model predictive control for nonlinear systems. **IEEE Trans. Automat. Control**, 2022.

[9] Johannes Köhler, Raffaele Soloperto, Matthias A Müller, and Frank Allgöwer. A computationally efficient robust model predictive control framework for uncertain nonlinear systems. **IEEE Trans. Automat. Control**, 2020.

[10] Andras Sasfi, Melanie N Zeilinger, and Johannes Köhler. Robust adaptive MPC using control contraction metrics. **Automatica**, 2023.

[11] Pan Zhao, Arun Lakshmanan, Kasey Ackerman, Aditya Gahlawat, Marco Pavone, and Naira Hovakimyan. Tube-certified trajectory tracking for nonlinear systems with robust control contraction metrics. **IEEE Robotics and Automation Letters**, 2022.

[12] Johannes Köhler, Matthias A Müller, and Frank Allgöwer. A novel constraint tightening approach for nonlinear robust model predictive control. In **Proc. American Control Conf. (ACC)**, 2018.

[13] Kim Peter Wabersich and Melanie N Zeilinger. A predictive safety filter for learning-based control of constrained nonlinear dynamical systems. **Automatica**, 2021.

References and further reading III

[14] D Limon, T Alamo, and EF Camacho. Input-to-state stable MPC for constrained discrete-time nonlinear systems with bounded additive uncertainties. In **Proc. 41st IEEE Conf. Decision and Control (CDC)**, 2002.

[15] Gilberto Pin, Davide M Raimondo, Lalo Magni, and Thomas Parisini. Robust model predictive control of nonlinear systems with bounded and state-dependent uncertainties. **IEEE Trans. Automat. Control**, 2009.

[16] Johannes Köhler, Peter Kötting, Raffaele Soloperto, Frank Allgöwer, and Matthias A Müller. A robust adaptive model predictive control framework for nonlinear uncertain systems. **Int. J. Robust and Nonlinear Control**, 2020.

[17] D Limon, JM Bravo, T Alamo, and EF Camacho. Robust MPC of constrained nonlinear systems based on interval arithmetic. **IEE Proceedings-Control Theory and Applications**, 2005.

[18] Boris Houska and Mario E Villanueva. Robust optimization for MPC. Springer, 2019.

[19] Andrea Zanelli, Jonathan Frey, Florian Messerer, and Moritz Diehl. Zero-order robust nonlinear model predictive control with ellipsoidal uncertainty sets. In **Proc. NMPC Conference**, 2021.

[20] Mario E Villanueva, Rien Quirynen, Moritz Diehl, Benoît Chachuat, and Boris Houska. Robust MPC via min–max differential inequalities. **Automatica**, 2017.

References and further reading IV

[21] Florian Messerer and Moritz Diehl. An efficient algorithm for tube-based robust nonlinear optimal control with optimal linear feedback. In **Proc. 60th IEEE Conference on Decision and Control (CDC)**, 2021.

[22] Antoine P Leeman, Johannes Köhler, Andrea Zanelli, Samir Bennani, and Melanie N Zeilinger. Robust nonlinear optimal control via system level synthesis. **arXiv**, 2023.

[23] Johannes Köhler, Elisa Andina, Rafaelle Soloperto, Matthias A Müller, and Frank Allgöwer. Linear robust adaptive model predictive control: Computational complexity and conservatism. In **Proc. 58th IEEE Conf. Decision and Control (CDC)**, 2019.

[24] Shaoru Chen, Victor M Preciado, Manfred Morari, and Nikolai Matni. Robust model predictive control with polytopic model uncertainty through system level synthesis. **arXiv**, 2022.