Modeling buildings from physics

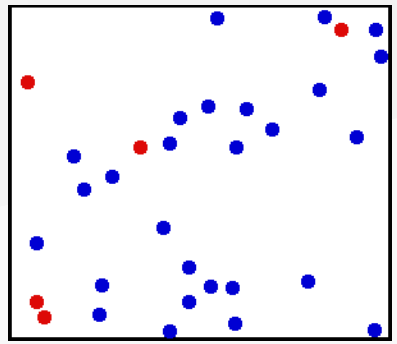
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Temperature (reminder)

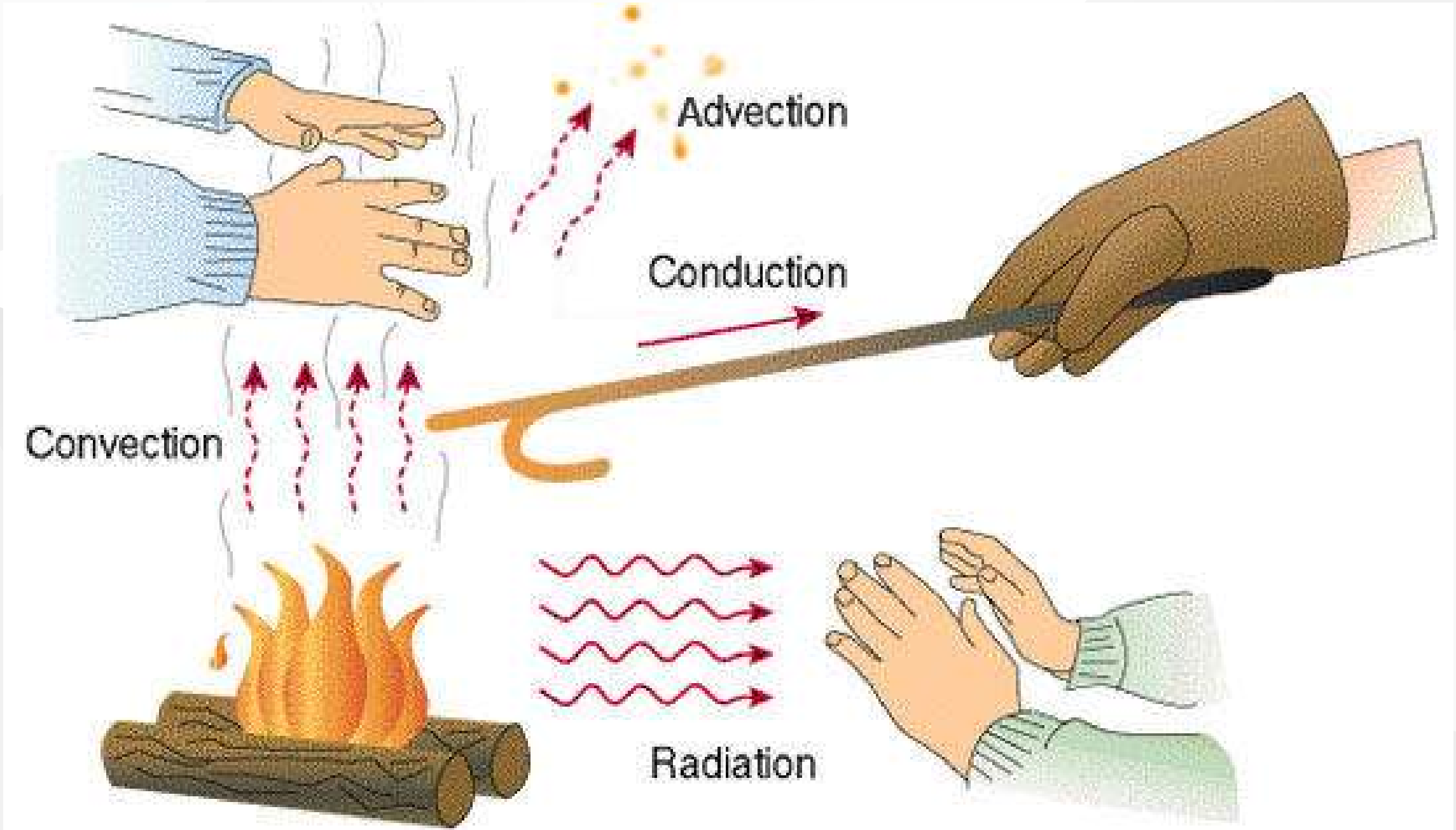
It's the **kinetic energy** of particles:

$$T = \frac{1}{3k_B} \int (\vec{v} - \vec{V})^2 p(\vec{v}) d\vec{v}$$



with k_B the Boltzmann constant, $\vec{v}$ the speed of a particle, $p(\vec{v})$ the density of probability of particle speeds and $\vec{V}$ the average speed (meaningful only if particle density enough)

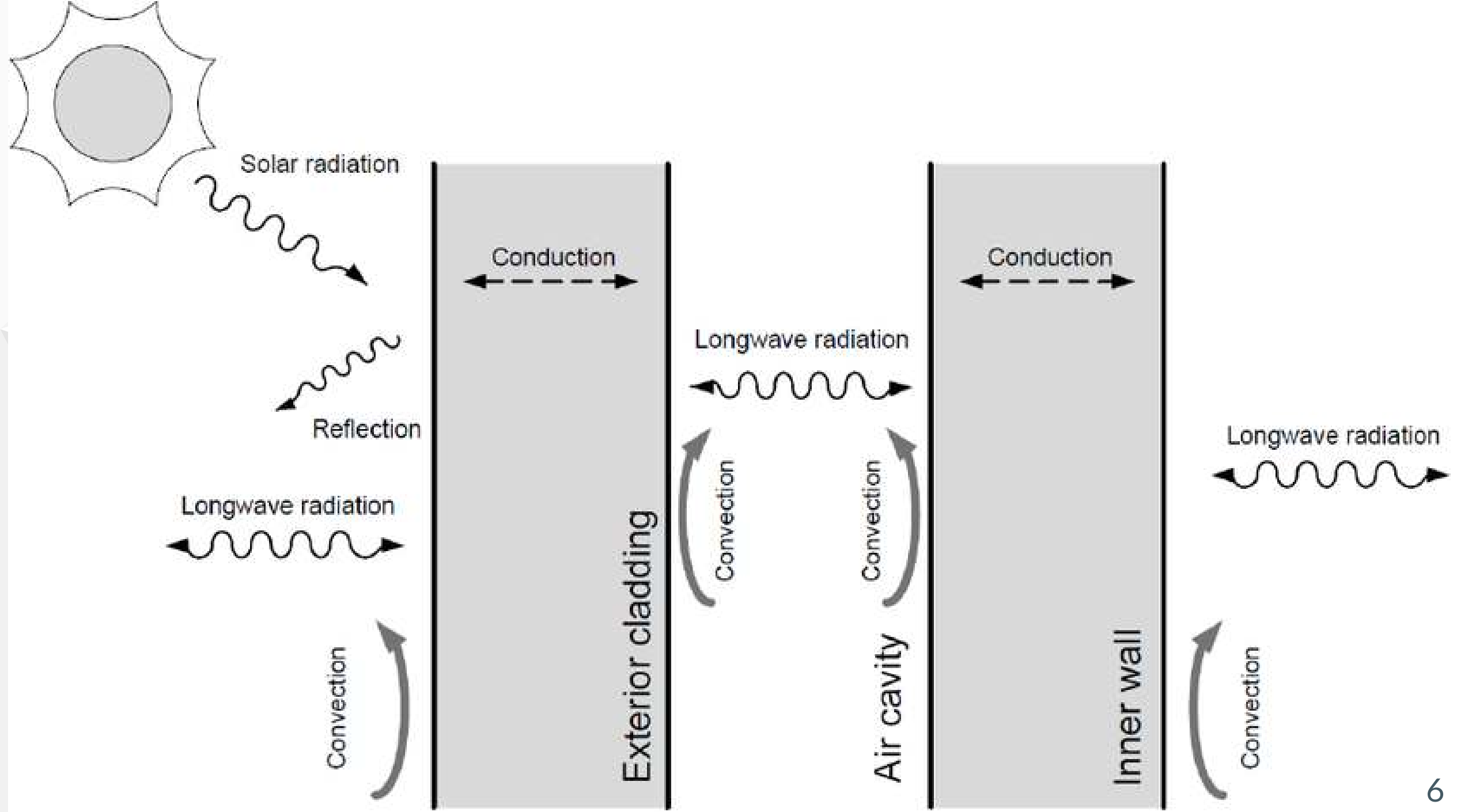
$$T_{\text{Kelvin}} = 273.15 + T_{\text{Celsius}} \qquad \left(T_{\text{Celsius}} = 32 + \frac{9}{5} T_{\text{Fahrenheit}} \right)$$


Advection
Conduction
Convection
Radiation

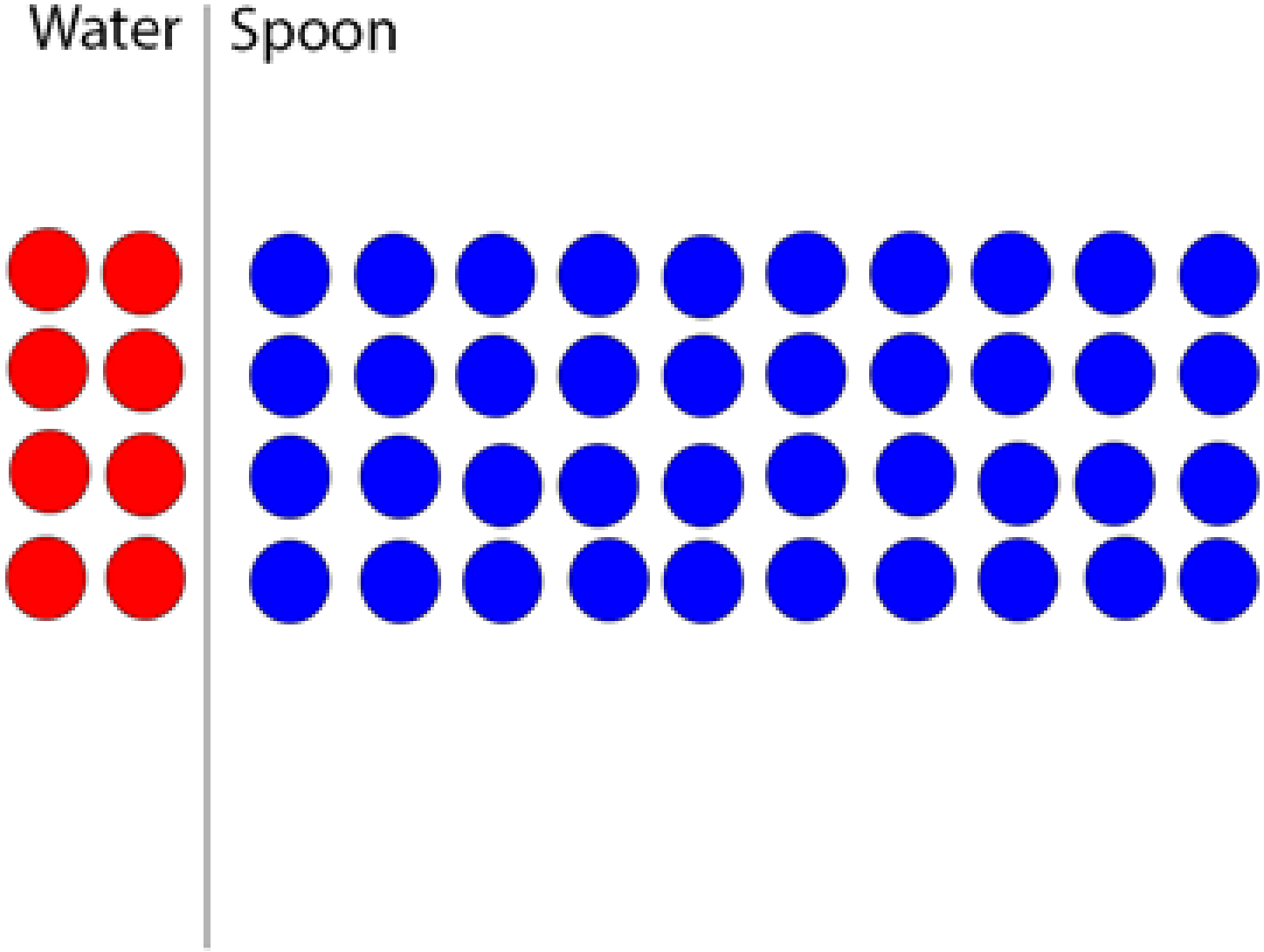
The next figure shows the different heat transfers affecting a shelter: conduction, convection, radiation and heat storage.

- the conduction corresponds to the transfer of molecular agitation through walls, slabs, roofs and other layers; it appears inside solids
- the convection corresponds to the transfer of heat by the movement of air or fluids; it appears inside gases and fluids close to surfaces where molecules are slow down because of the viscosity

- the radiation corresponds to the transfer of heat by electromagnetic waves: it appears at the surface of opaque materials via emission,reflection, absorption and transmission inside transparent materials
- the advection (not represented here) corresponds to the transfer of heat by the movement of air or fluids, it appears in air conditioning systems with ventilation or infiltration.
- the heat storage (not represented here) corresponds to the pure controllable storage of heat within a materials or a liquid.


Solar radiation
Reflection
Longwave radiation
Convection
Conduction
Exterior cladding
Longwave radiation
Convection
Air cavity
Conduction
Inner wall
Longwave radiation
Convection

Conduction within solids



Sun emits short wave radiations because it is at a very high temperature (5778 K). Depending on the surface these radiations hit, they are either reflected, transmitted or absorbed. Because absorption yields an increase of the surface material temperature, short wave radiations are converted into long wave radiations which are exchanged between walls, roofs and even with the sky. The walls and roofs emit long wave radiations (terrestrial temperature) but most glazing are opaque to long wave radiations.

- the heat storage corresponds to the storage of heat within a materials forming inertia or pure storage

To make it easy, a simplified approach is proposed by the ThBAT (official RT2012 and RE2020 method for thermal calculation), with typical resistance values for convection and radiation valid in most situations.

Nevertheless, the physics from which it comes from will be developed:

1. to figure out the underlying assumptions
2. to be able to extend to special cases

The general isotropic heat conduction equation is given by:

$$\mathrm{div}(\overrightarrow{\mathrm{grad}(T(X,t))}) + \frac{1}{\lambda}Q(X,t) = \frac{\rho c}{\lambda}\frac{\partial T(X,t)}{\partial t}$$

with ρ, the density (kg/m^2), λ the conductivity (W/m.K), c the material specific heat capacity (J/kg.K), T(X,t) the temperature (K) at position X and time t and $Q(X,t)$ the heat power.

$\frac{\rho c}{\lambda}$ is the **thermal diffusivity**: the larger, the faster the heat will diffuse. The **volumetric heat storage capacity** is given by: ρc

material	conductivity (W/m.K)	specific heat capacity (J/kg.K)	density (kg/m3)	emmisivity coefficient	thermal diffusivity (mm^2/s)	volumetric heat capacity (J/m3/K)
immobile CO2	0.01	1,005	1.56	0	8.92	1570
immobile air	0.02	1,005	1.26	0	19.03	1261
polyurethane foam	0.03	1,000	30	0.82	0.97	30000
dry glass wool	0.05	840	25	0.97	2.38	21000
asbestos (amiante)	0.15	816	2,400	0.93	0.08	1958400
lightweight wood	0.04	1,300	400	0.95	0.08	520000
heavyweight wood	0.12	2,400	890	0.95	0.06	2136000
plaster	0.35	1,090	2,300	0.98	0.14	2507000
water	0.60	4,180	997	0.95	0.14	4167460
glass	1	753	2,600	0.94	0.51	1957800
full brick	1	840	2,000	0.93	0.60	1680000
concrete	0.5	880	2,000	0.85	0.28	1760000

material	conductivity (W/m.K)	specific heat capacity (J/kg.K)	density (kg/m3)	emmisivity coefficient	thermal diffusivity (mm^2/s)	volumetric heat capacity (J/m3/K)
graphite	5	780	2,500	0.96	2.56	1950000
stainless steel	15	440	7,820	0.07	4.36	3440800
lead	35	129	11,350	0.43	23.90	1464150
iron	57	449	7,200	0.44	17.63	3232800
aluminium	203	897	2,700	0.07	83.82	2421900
copper	380	385	8,790	0.03	112.29	3384150
silver	410	235	10,500	0.03	166.16	2467500
wood	0.20	1,300	700	0.95	0.22	910000
PVC	0.25	1,046.00	1,395.00	0.92	0.17	1459170

For more materials, see `/buildingenergy/data/propertiesDB.xslx`

The Fourier empirical law provides a solution:

$$\overrightarrow{\varphi(X,t)} = -\lambda \overrightarrow{grad(T(X,t))}$$

Introducing the Fourier law into the general isotropic heat conduction equation, gives:

$$\rho c \frac{\partial T(X,t)}{\partial t} = -\mathrm{div}(\overrightarrow{\varphi(X,t)}) + Q(X,t)$$

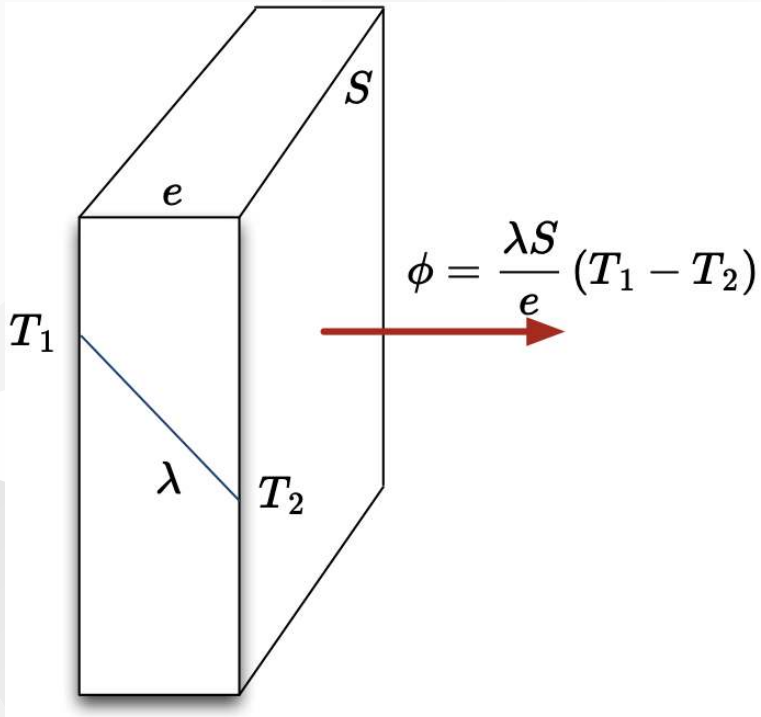
In steady state ($t = t_\infty$) without internal heat, it yields:

$$\mathrm{div}(\overrightarrow{\varphi(X,t)}) = 0 \rightarrow \overrightarrow{\varphi(X,t)} = \Phi$$

For a very small element: $\Phi = -\lambda \begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial x} \end{bmatrix}$

For a single layer wall (sum up over a close area) of surface S and thickness e:

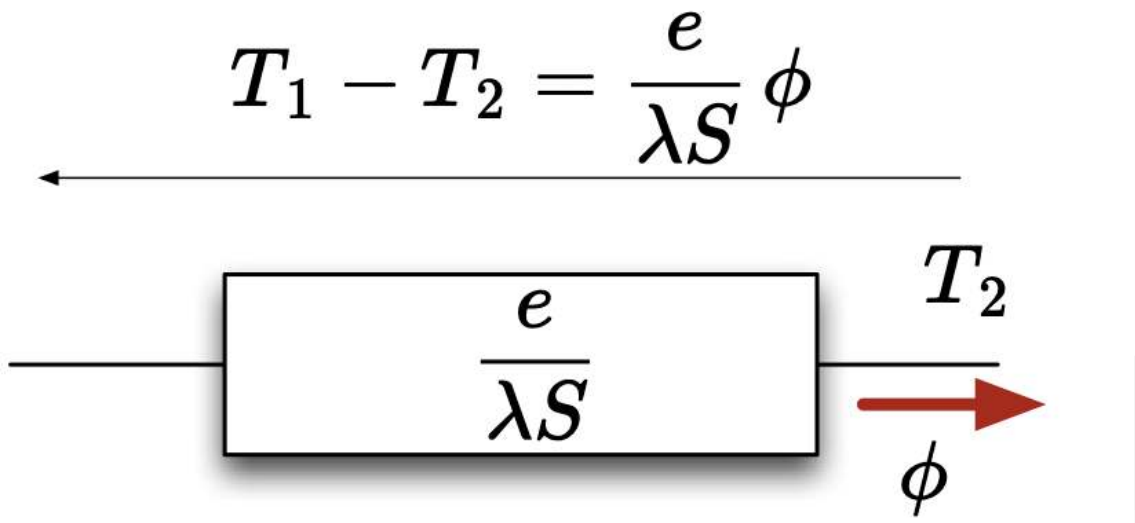
$$\phi = \frac{\lambda S}{e}(T_1 - T_2)$$



The transmitivity coefficient is: $U = \frac{\lambda}{e}$ in $W/m^2.K$

The thermal resistance is $R = \frac{e}{\lambda}$ in $m^2.K/W$ or $R = \frac{e}{\lambda S}$ in K/W

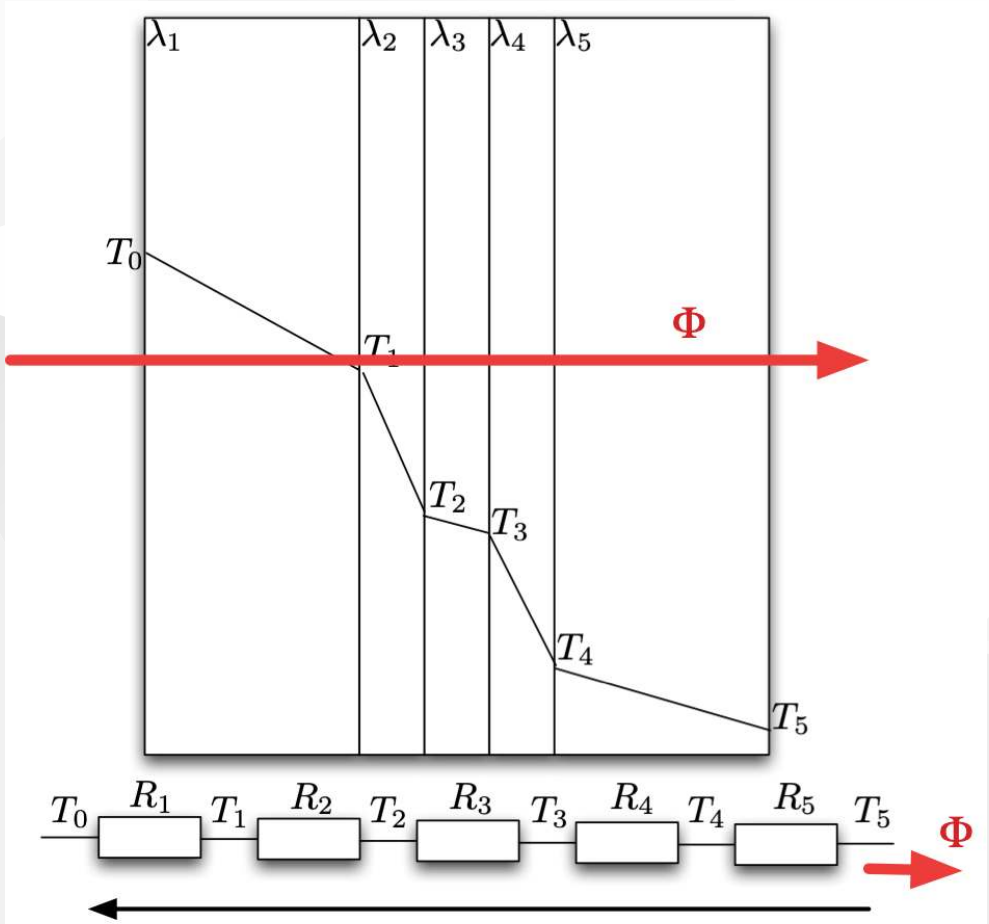
A single layer can be modeled by a electrical resistance where temperatures are equivalent to voltages and currents to heat flows:



Remark: Because of the minus sign, the first term of the temperature subtraction is at the opposite side of the heat flow direction.

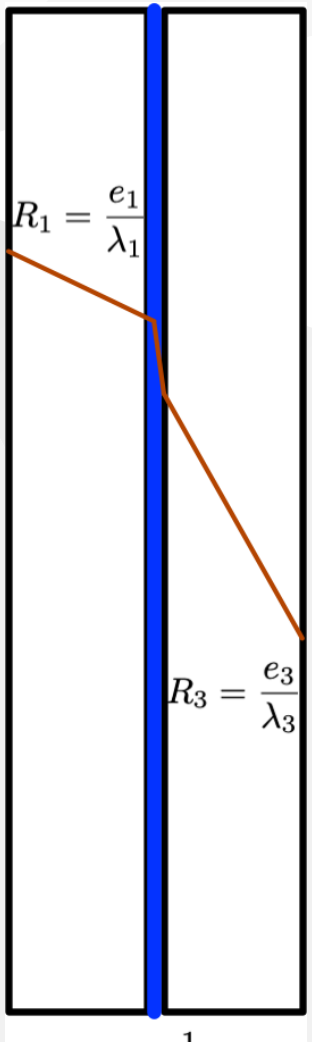
It's named a nodal model: each node = a temperature.

Let's consider a multi-layer wall:



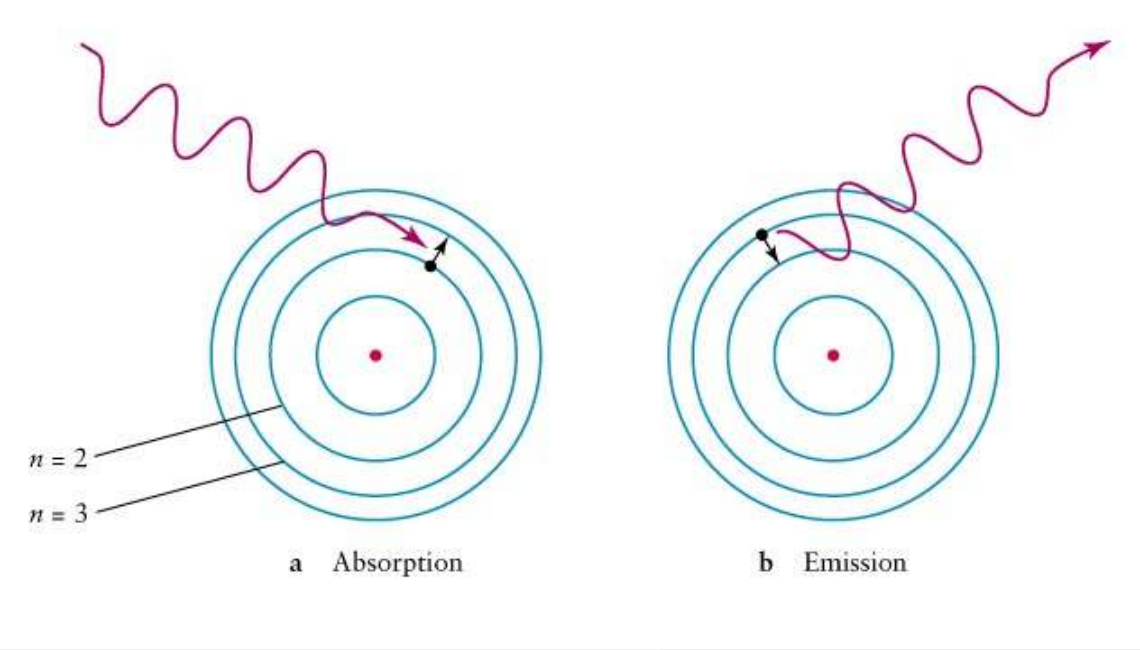
$$T_0 - T_5 = (T_0 - T_1) + (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) + (T_4 - T_5)$$
$$= \left(\frac{e_1}{\lambda_1 S} + \frac{e_2}{\lambda_2 S} + \frac{e_3}{\lambda_3 S} + \frac{e_4}{\lambda_4 S} + \frac{e_5}{\lambda_5 S} \right) \Phi$$

Sometimes, it could be discontinuity in the temperature variation: it's modelled by contact resistances:



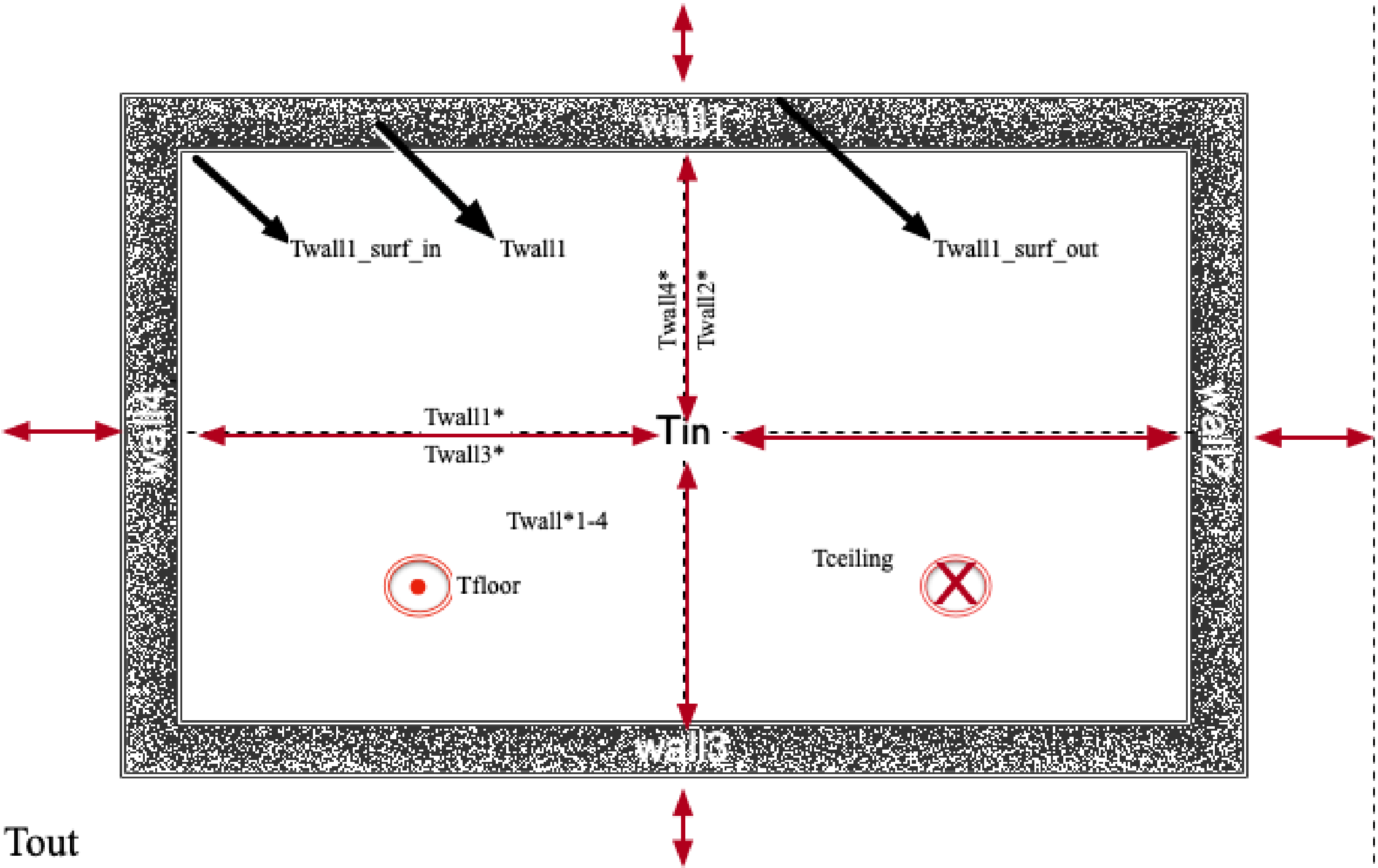
Modeling radiative exchanges between walls

As seen before, every surface made of a given material at a given temperature (in Kelvin for radiative exchanges) is behaving more or less like a black body i.e. a wall is emitting and exchanging power with the other walls.



Different approaches more or less accurate but also, more or less complex, can be adopted. The most complex and accurate is to compute shape factor of each surface relatively to the other i.e. how a surface is visible by another but this leads to complex calculations (some tables gathering common configurations can be found in the scientific literature).

An easier way is to consider each wall is exchanging with a virtual wall facing it at indoor air temperature, or with a virtual wall at outdoor air temperature (or at sky temperature), having the same emissivity.

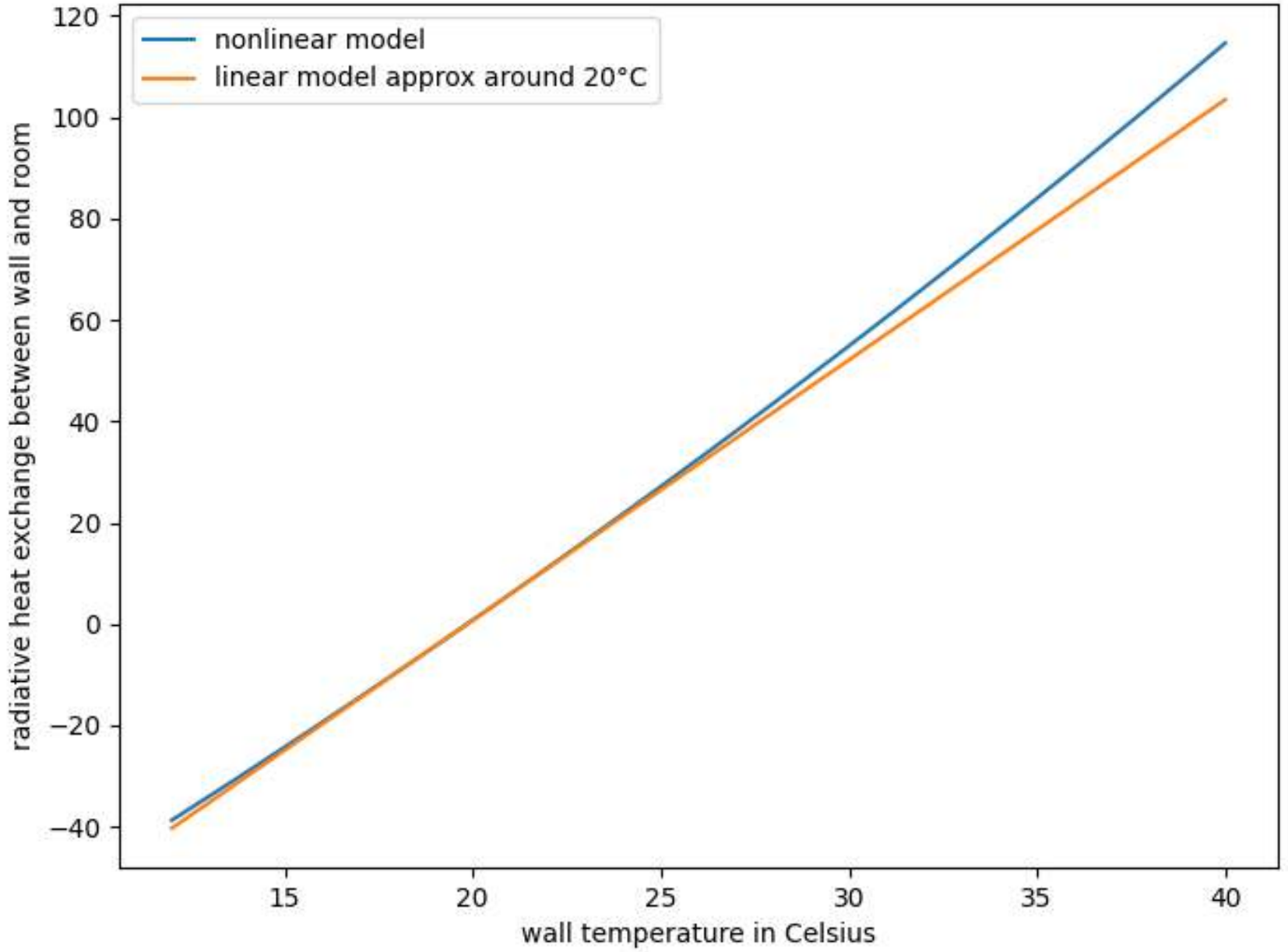

Twall2
Twall1_surf_in
Twall1
Twall1_surf_out
Twall4*
Twall2*
Twall4
Twall2
Twall1*
Tin
Twall3*
Twall*1-4
Tfloor
Tceiling
Twall3
Tout

Instead of calculated the radiative heat directly exchange between wall 1 and wall 3 (for example), one computes the heat exchange between wall 1 and the virtual wall $wall_1^*$ at a indoor air temperature T_{in} and similarly, between wall 3 and the virtual wall $wall_3^*$ at a indoor air temperature T_{in}.

$$\varphi_1 = \sigma\epsilon_1 T_1^4 - \sigma\epsilon_1 T_{in}^4 \approx 4\sigma\epsilon_1 \bar{T}_K^3 (T_1 - T_{in}) = h_{r,1}(T_1 - T_{in})$$

with:

- $\bar{T}_K$, an average temperature in Kelvin ($T_K = T_C + 273.15^\circ C$)
- $h_{r,1} = 4\sigma\epsilon_1 \bar{T}_K^3$, the radiative heat transfer coefficient for wall 1
- σ, the Stefan-Boltzmann constant: $5.670374419 \times 10^{-8} W/m^2.K^4$


nonlinear model
linear model approx around 20°C
radiative heat exchange between wall and room
wall temperature in Celsius
120
100
80
60
40
20
0
−20
−40
15
20
25
30
35
40

As a summary, we have, for each wall with a surface temperature T_1:

$$\varphi_1 = h_{r,1}(T_1 - T_{in})$$

with:

- $\bar{T}_K$, an average temperature in Kelvin
  ($\bar{T}_K = 293K$ for indoor, $\bar{T}_K = 286K$ for outdoor)
- $h_{r,1} = 4\sigma\epsilon_1\bar{T}_K^3$, the radiative heat transfer coefficient for wall 1
- σ, the Stefan-Boltzmann constant: $5.670374419 \times 10^{-8} W/m^2.K^4$

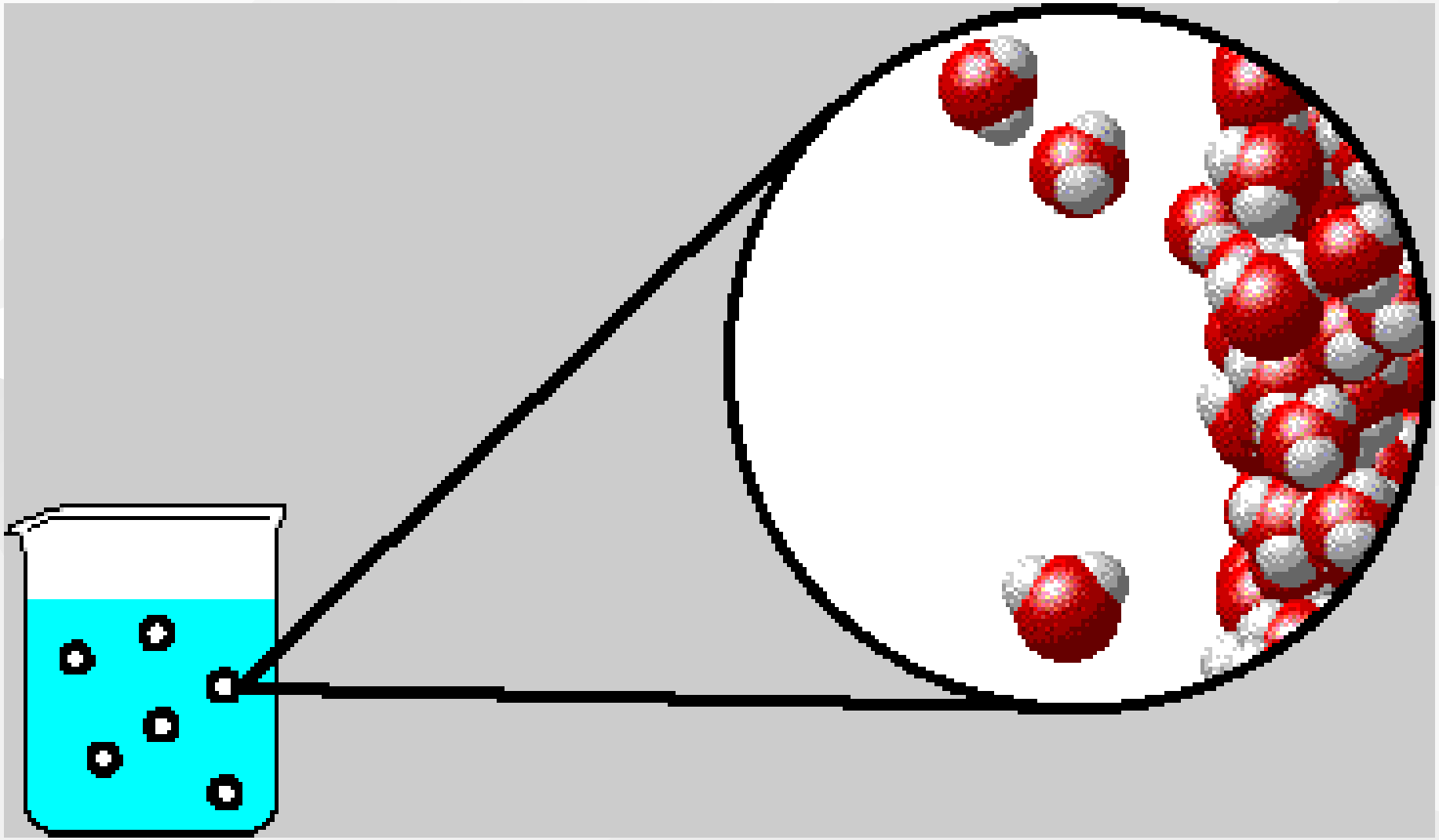
Reminder from "slides01_sun.md":

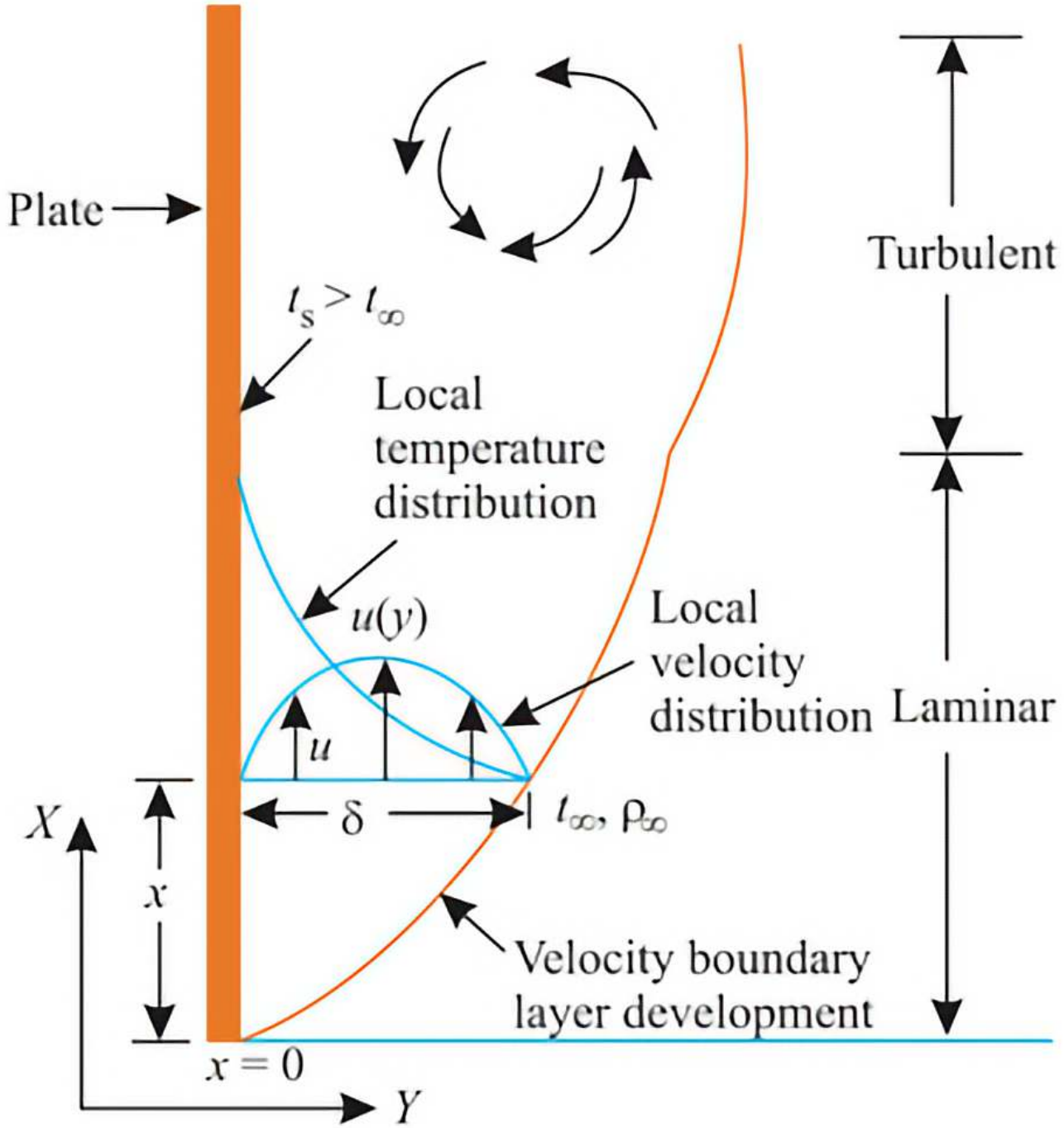
Few emissivity values are given below. The file propertiesDB.xlsx provide much more values.


image-20240428005608289

Convection

When a surface is in contact with a gas, in particular air, at a temperature higher or lower than that of the surface, a layer of gas adjacent to the surface gets heated or cooled. A density difference is created between this layer and the surface surrounding it. The density difference introduces a buoyant (poussée d'Archimède) force causing flow of gas near the surface. Heat transfer under such conditions is known as **free or natural convection**. Thus “Free or natural convection is the process of heat transfer which occurs due to movement of the fluid particles by density changes associated with temperature differential in a fluid.”




Plate
$t_s > t_\infty$
Local
temperature
distribution
$u(y)$
Local
velocity
distribution
u
δ
t_∞, ρ_∞
x
X
Y
$x = 0$
Velocity boundary
layer development
Turbulent
Laminar

In building heat transfer, convection is the process by which heat is transferred through the movement of air or fluid within the building. Convection occurs in two main forms: natural (or free) convection and forced convection.

1. Natural Convection relies on temperature differences within the air. When air is heated, it becomes less dense and rises, while cooler, denser air descends. This creates a circulation pattern.

Example: In a heated room, air near a radiator or heating source warms up, rises to the ceiling, and spreads across the room. As it cools, it falls, allowing warmer air to take its place, creating a loop.

2. Forced Convection occurs when fans, blowers, pumps or simply wind actively move air across surfaces, improving heat transfer by continuously replacing air near surfaces with air at a different temperature.

Example: HVAC systems use fans to push warm or cool air through ducts, delivering it to different areas within a building. This forced circulation accelerates the heat transfer process and maintains consistent indoor temperatures.

Convection along surfaces

Immobile air does not exist because of convection i.e. conduction only applies to solid: the conductivity value (λ) can not be used directly as for solids. Insulation is a material made of lots of small alveoli to immobilize air. Therefore, all the insulation materials have a λ close but higher than the λ of the immobile air (λ_{air}=0.024W/mK @0°C, λ_{air} =0.025W/mK @20°C).

The model use is named Newton's law:

$$\varphi_{\text{conv}} = h_c S(T_{\text{surf}} - T_{air})$$

where h_c is named convective heat transfer coefficient.

For natural convection,

$$h_c = \frac{\lambda N_u}{L}$$

with

- $N_u = CR_a^n$, the Nusselt dimensionless number, whose constant C and n depends on the problem to solve (for instance, $C = 0.59$ and $n = 0.25$),
- λ, the thermal conductivity of air in W/m.K
- R_a, the Rayleigh dimensionless number for air
- L, a typical length, for instance, the height of a wall

The Rayleigh number for air in a typical indoor environment can vary widely depending on factors such as the temperature difference between the wall and the air, the height of the surface, and the properties of the air.

It is calculated like this:

$$R_a = \frac{g\beta(T_{ref} - T_{other})L^3}{\nu\alpha}$$

where:

...

- T_{ref} and T_{other} are 2 temperatures of interest depending on the problem to solve (wall, cavity,...)
- g is the acceleration due to gravity (9.81 m/s²)
- β is the thermal expansion coefficient of air ($\approx 1/\bar{T}_K$) with $\bar{T}_K$ an average temperature in Kelvin.
- L is a typical height in the problem to solve
- ν is the kinematic viscosity of air
- α is the thermal diffusivity of air

Kinematic viscosity (in m^2/s) of air depends on temperature. It can be deduced from:

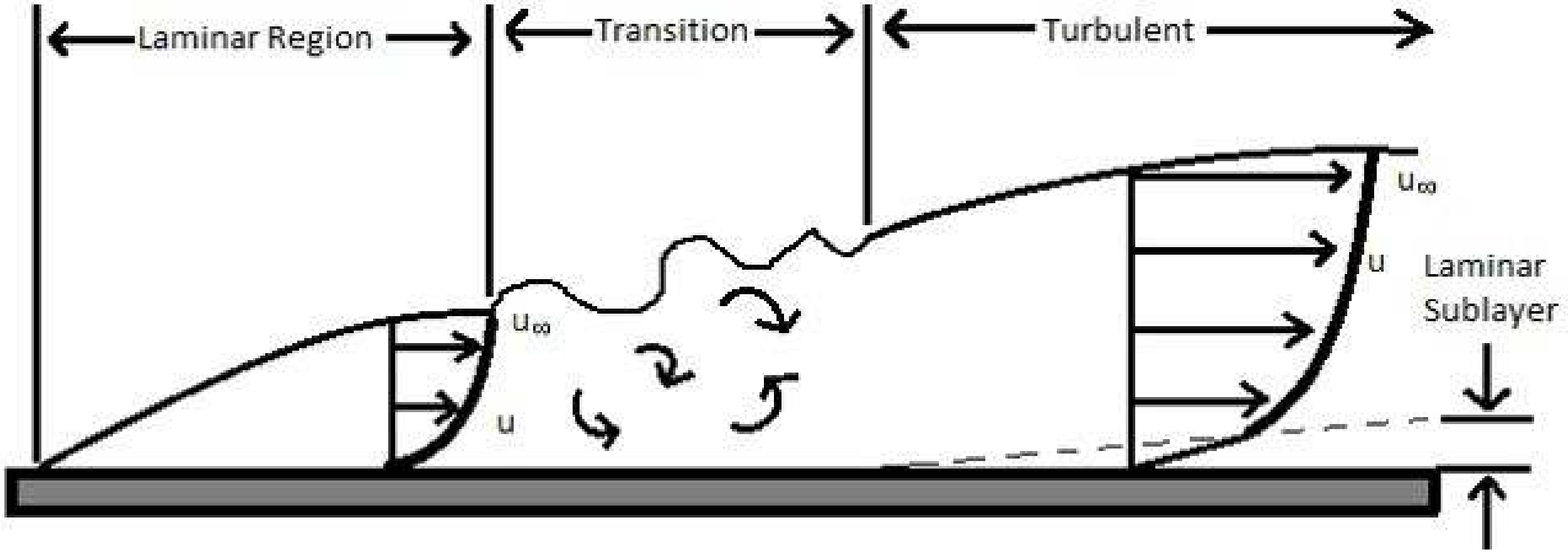
T(°C)	-25	-15	-10	-5	0	5	10
ν	11.18e-6	12.01e-6	12.43e-6	12.85e-6	13.28e-6	13.72e-6	14.16e-6

Thermal diffusivity (in m^2/s too) of air depends also on temperature:

T(°C)	-53.2	-33.2	-13.2	0	6.9	15.6	26.9	46.9
α	12.6e-6	14.9e-6	17.3e-6	19e-6	19.9e-6	21e-6	22.6e-6	25.4e-6

Thermal conductivity (in $W/m.K$) of air also depends on temperature:

T(°C)	-50	-25	-15	-10	-5	0	5
λ	20.41e-3	22.41e-3	23.20e-3	23.59e-3	23.97e-3	24.36e-3	24.74e-3


Laminar Region
Transition
Turbulent
u∞
u
u∞
u
Laminar
Sublayer

The Rayleigh number is the ratio of thermal transport via diffusion wrt thermal wrt thermal transport via convection:

- $R_a < 10^3$, air is not moving and the conduction is dominating
- $10^3 \leq R_a \leq 10^9$, air is behaving mainly as a laminar flow
- $R_a > 10^9$, air is behaving mainly as a turbulent flow

The Nusselt number is the ratio of total heat transfer to conductive heat transfer at a boundary. It determines whether a flow condition is rather immobile, laminar or turbulent:

- $N_u \leq 1$, conduction mainly (air is immobile)
- $1 < N_u < 100$, laminar flow mainly
- $N_u > 100$, turbulent flow mainly

Free convection along a vertical (indoor) wall

$$R_c = \frac{1}{h_c S}$$

with S is the wall surface, and

$$h_c = \frac{\lambda N_u}{L}$$

or, with radiative transfer, the thermal resistance of the whole boundary air layer:

$$R_L = \frac{1}{(h_c + h_r) S}$$

The Nusselt number is given by:

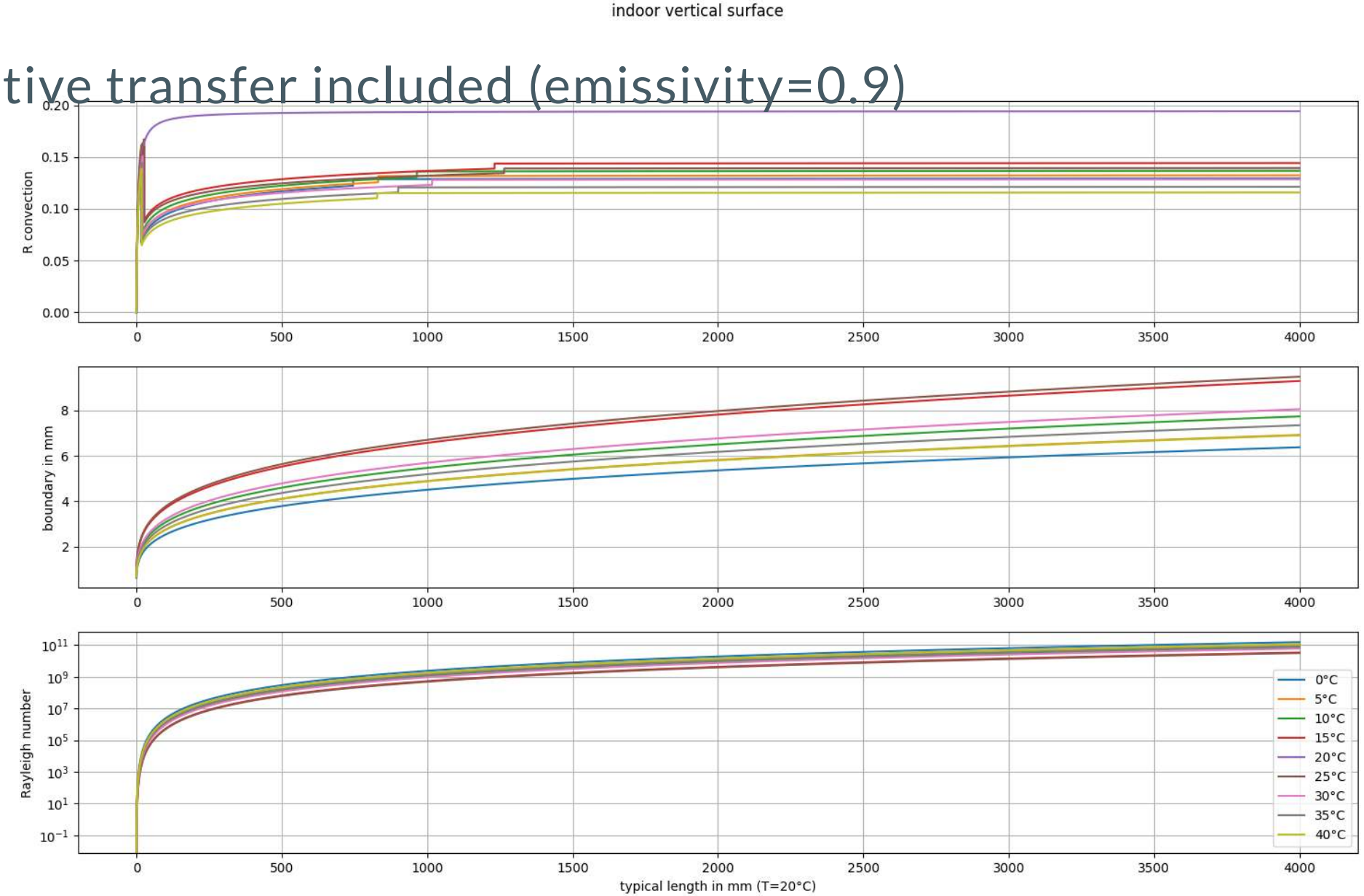
- $R_a \leq 10^4$, $N_u = 1$, conduction is dominating
- $10^4 < R_a < 10^9$, $N_u = 0.68 + \frac{0.67 R_a^{0.25}}{\left(1+\frac{0.492}{Pr}\right)^{0.25}}$, laminar flow mainly
- $R_a \geq 10^9$, $N_u = 0.1 R_a^{0.33}$, turbulent flow mainly

where $P_r \approx 0.71$ is the Prandtl number.

The thickness δ of the resulting air layer is given by:

$$\delta = \frac{L}{R_a^{0.25}}$$

Radiative transfer included (emissivity=0.9)



Free convection along a horizontal (indoor) floor

$$R_c = \frac{1}{h_c S}$$

with S is the floor surface, and $h_c = \frac{\lambda N_u}{L}$

With radiative transfer, the thermal resistance of the whole boundary air layer:

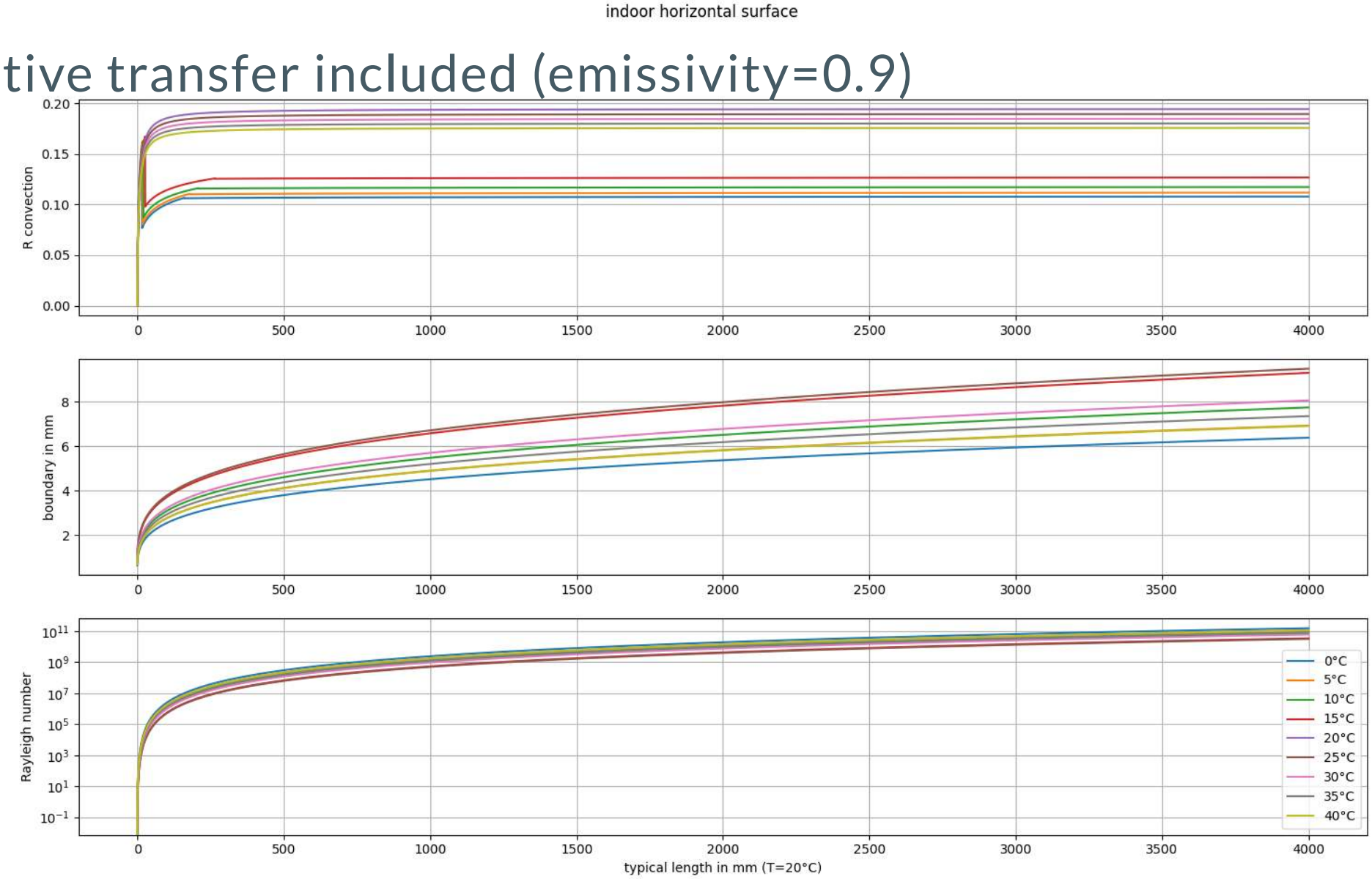
$$R_L = \frac{1}{(h_c + h_r)S}$$

For an ascending flow, the Nusselt number is given by:

- $R_a \leq 10^4$, $N_u = 1$, conduction is dominating
- $10^4 < R_a < 10^7$, $0.54 R_a^{0.25}$, laminar flow mainly
- $R_a \geq 10^7$, $N_u = 0.15 R_a^{0.33}$, turbulent flow mainly

For descending flow, $N_u = 1$

Radiative transfer included (emissivity=0.9)



Free convection inside a vertical cavity

$$R_c = \frac{1}{h_c S}$$

with S is the cavity surface, $h_c = \frac{\lambda N_u}{L}$ and $N_u = C{R_a}^n$

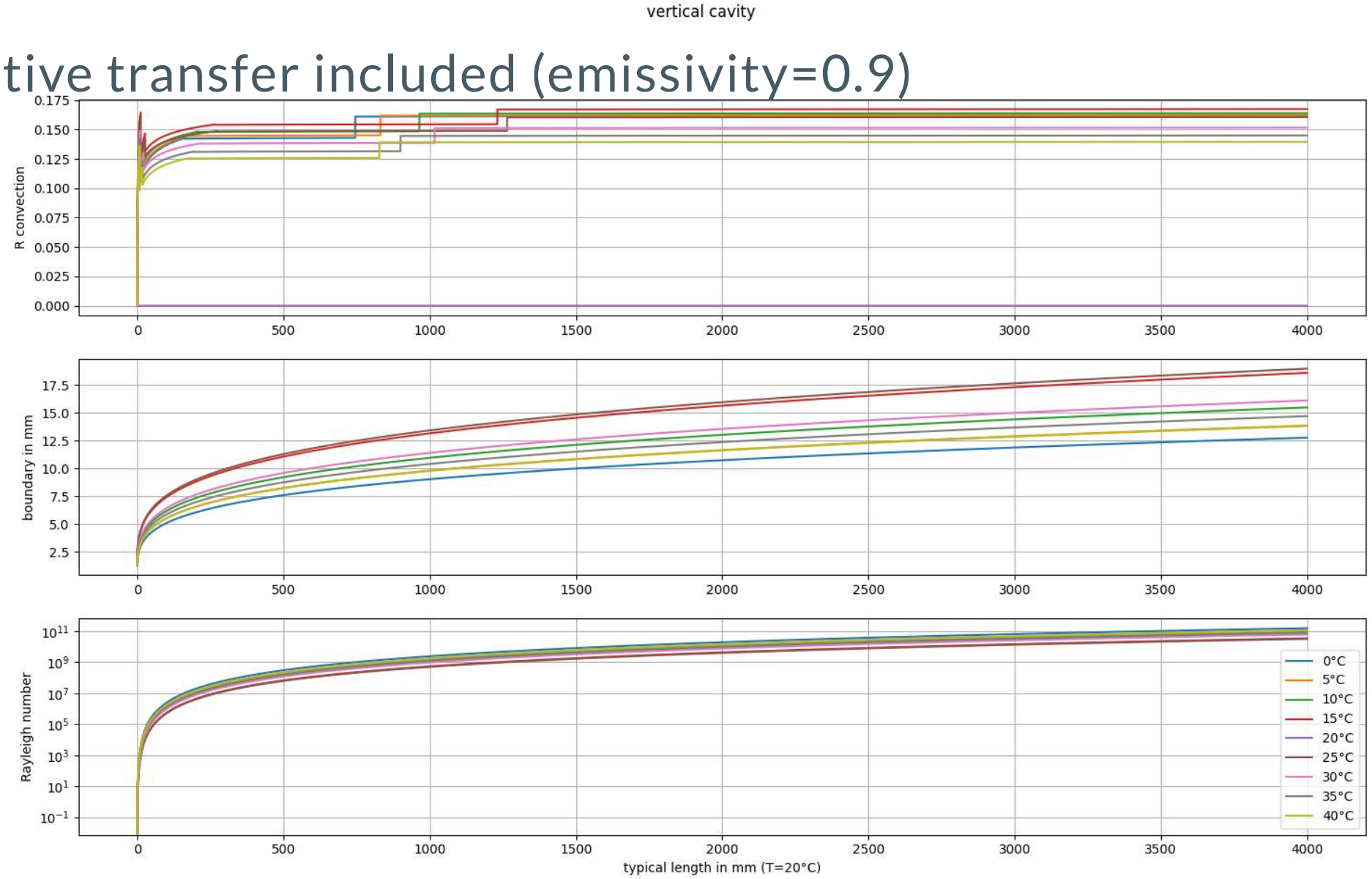
With radiative transfer, the thermal resistance of the whole boundary air layer:

$$R_L = \frac{1}{(h_c + h_r)S}$$

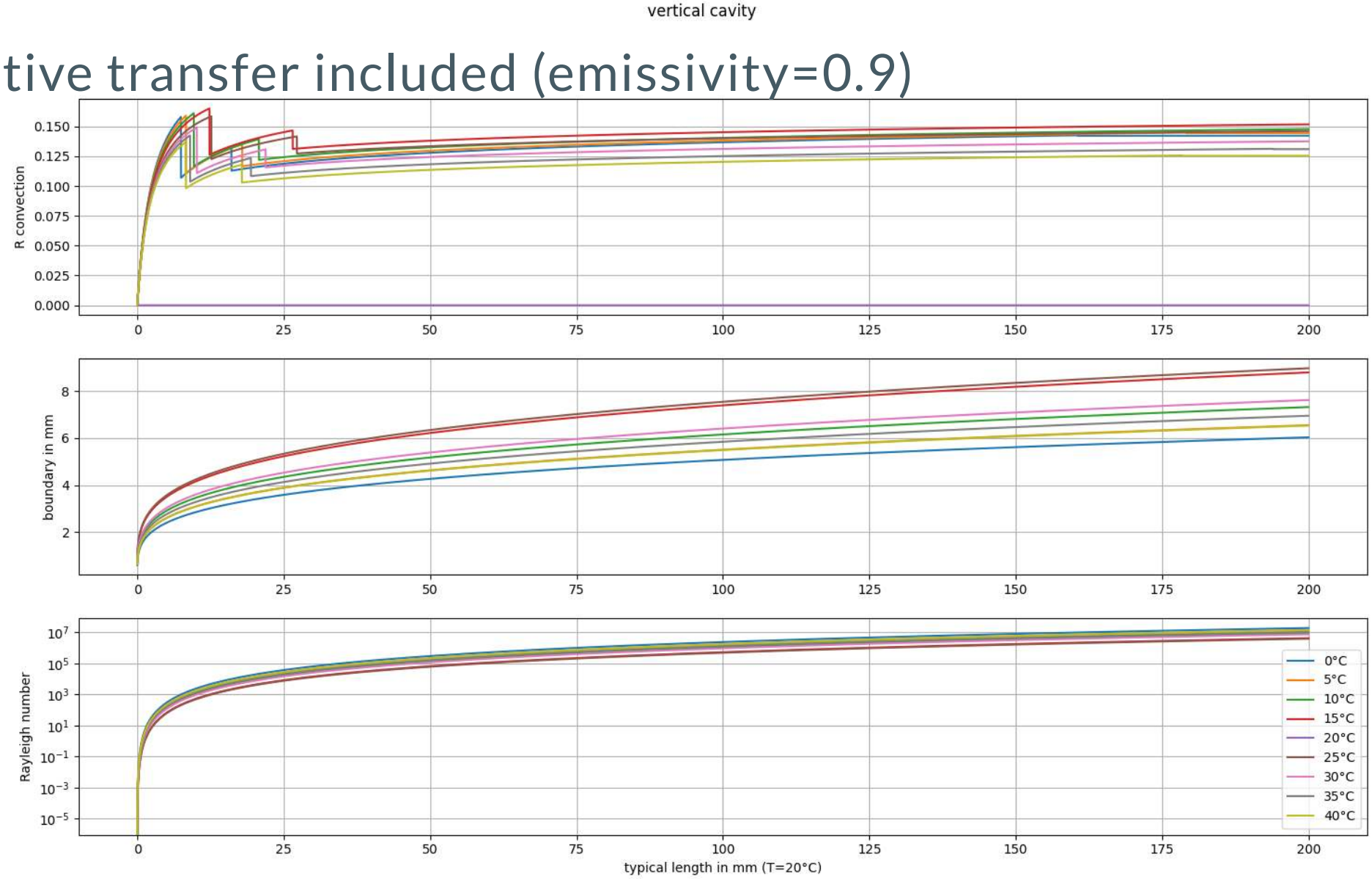
The coefficient C and n involved in the Nusselt number is given by:

- $R_a \leq 10^3$, $N_u = 1$, conduction is dominating
  laminar flow mainly
- $10^3 < R_a < 10^4, C = 1.18, n = 0.125$
- $10^4 < R_a < 10^7, C = 0.54, n = 0.25$
- $10^7 < R_a < 10^9, C = 0.15, n = 0.33$
  turbulent flow mainly
- $R_a \geq 10^9, C = 0.10, n = 0.33$

Radiative transfer included (emissivity=0.9)



Radiative transfer included (emissivity=0.9)



Free convection inside a horizontal cavity

$$R_c = \frac{1}{h_c S}$$

with S is the cavity surface, $h_c = \frac{\lambda N_u}{L}$ and $N_u = C{R_a}^n$

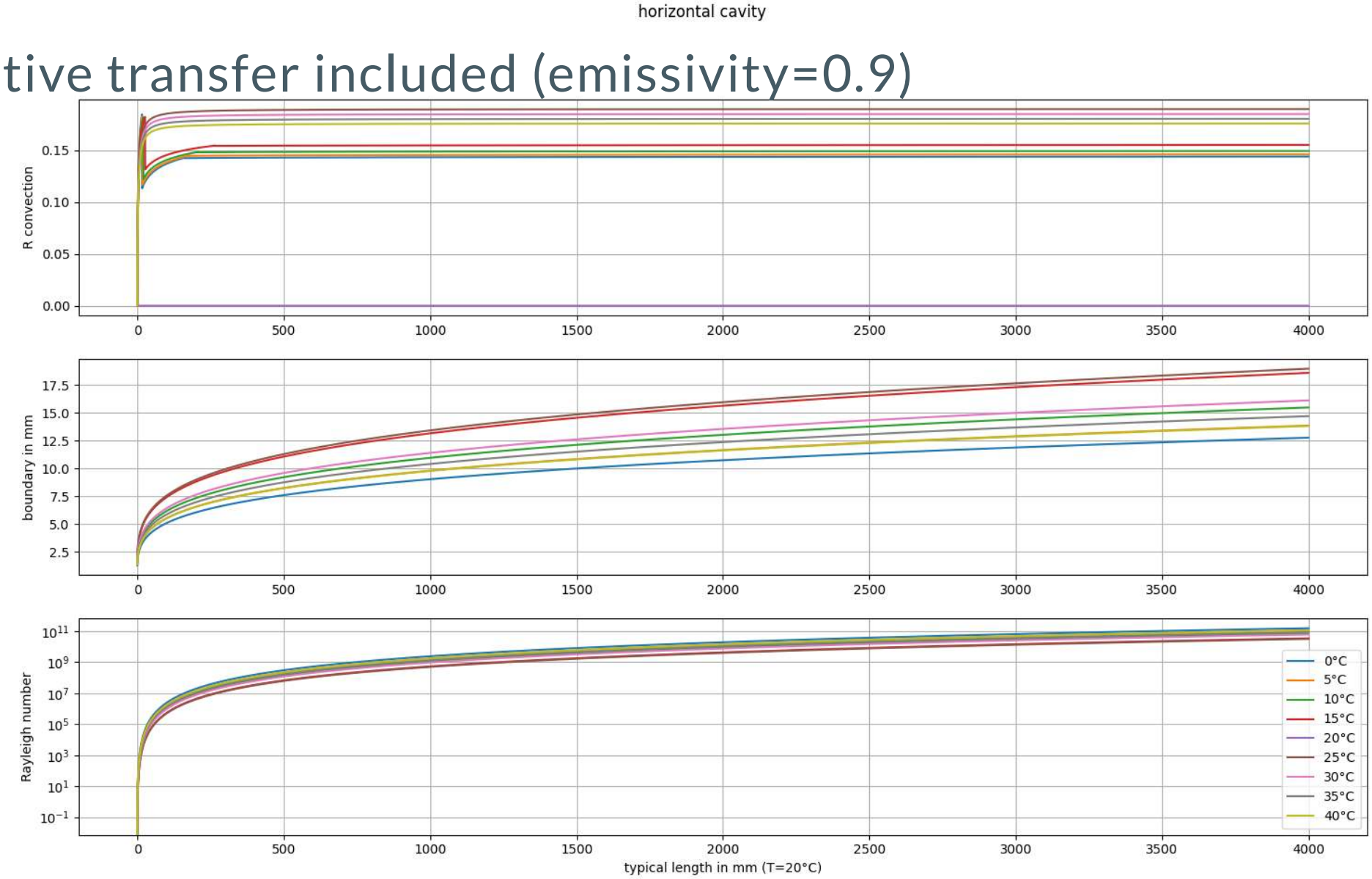
With radiative transfer, the thermal resistance of the whole boundary air layer:

$$R_L = \frac{1}{(h_c + h_r)S}$$

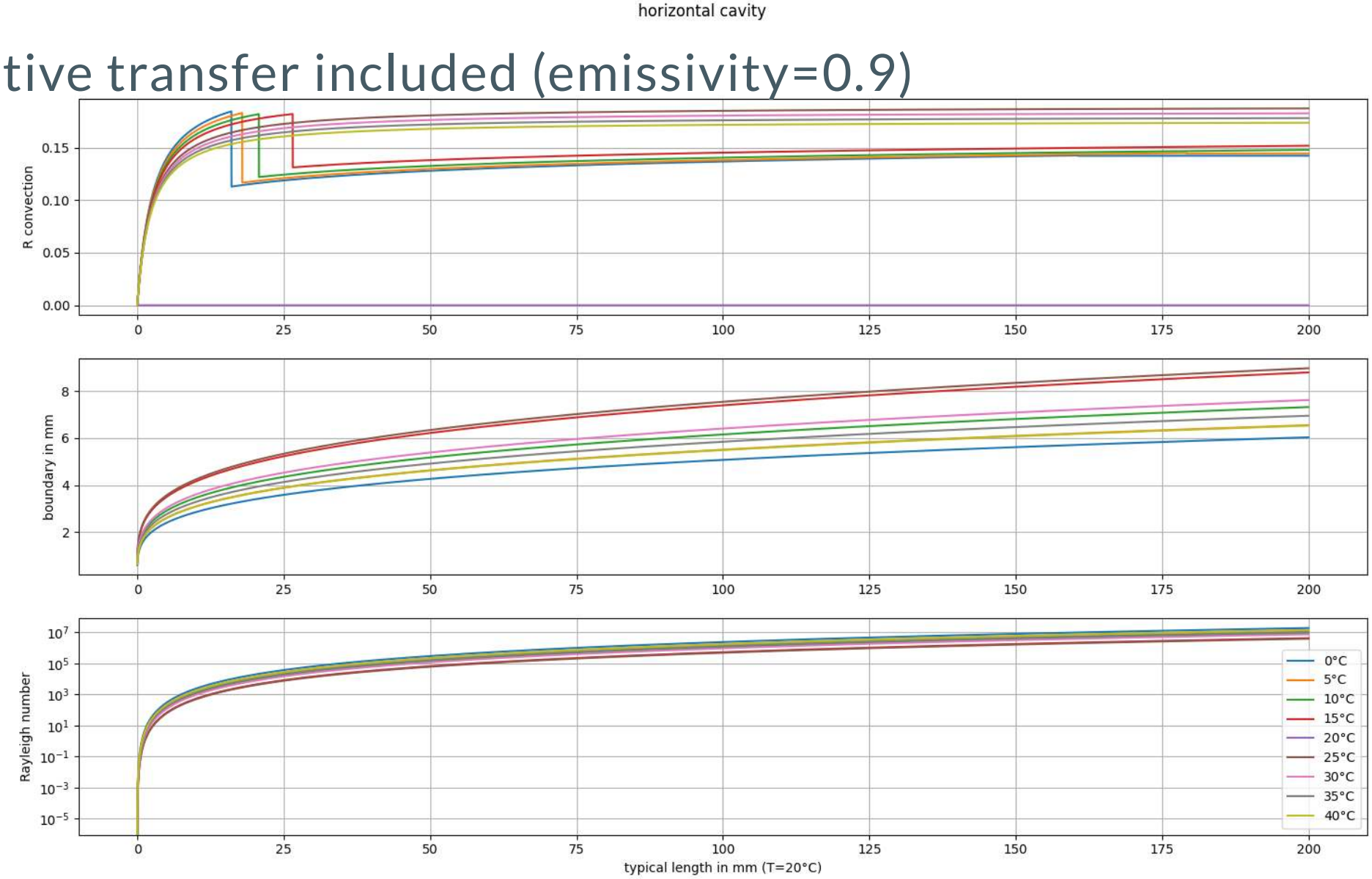
The Nusselt number is given by:
hot floor on top, conduction is dominating

- $R_a \leq 10^3, N_u = 1$
  laminar flow mainly
- $10^3 < R_a < 10^7, N_u = 0.54{R_a}^{0.25}$
  turbulent flow mainly
- $R_a \geq 10^7, N_u = 0.15{R_a}^{0.33}$

Radiative transfer included (emissivity=0.9)



Radiative transfer included (emissivity=0.9)



Outdoor, in the presence of wind, we satisfy the **forced convection** conditions. The Reynolds number (R_e) is used to predict the transition to laminar forces. It is a ratio between inertial and viscous forces:

- $R_e < 2300$, mainly laminar regime
- $2300 \leq R_e \leq 4000$, transition regime
- $R_e > 4000$, turbulent regime

Wind against a vertical wall

$$R_c = \frac{1}{h_c S}$$

with S is the wall surface, $h_c = \frac{\lambda N_u}{L}$ and $N_u = 0.037 R_e^{0.8} P_r^{0.33}$ ($P_r = 0.71$, the Prantl number)

With radiative transfer, the thermal resistance of the whole boundary air layer:

$$R_L = \frac{1}{(h_c + h_r) S}$$

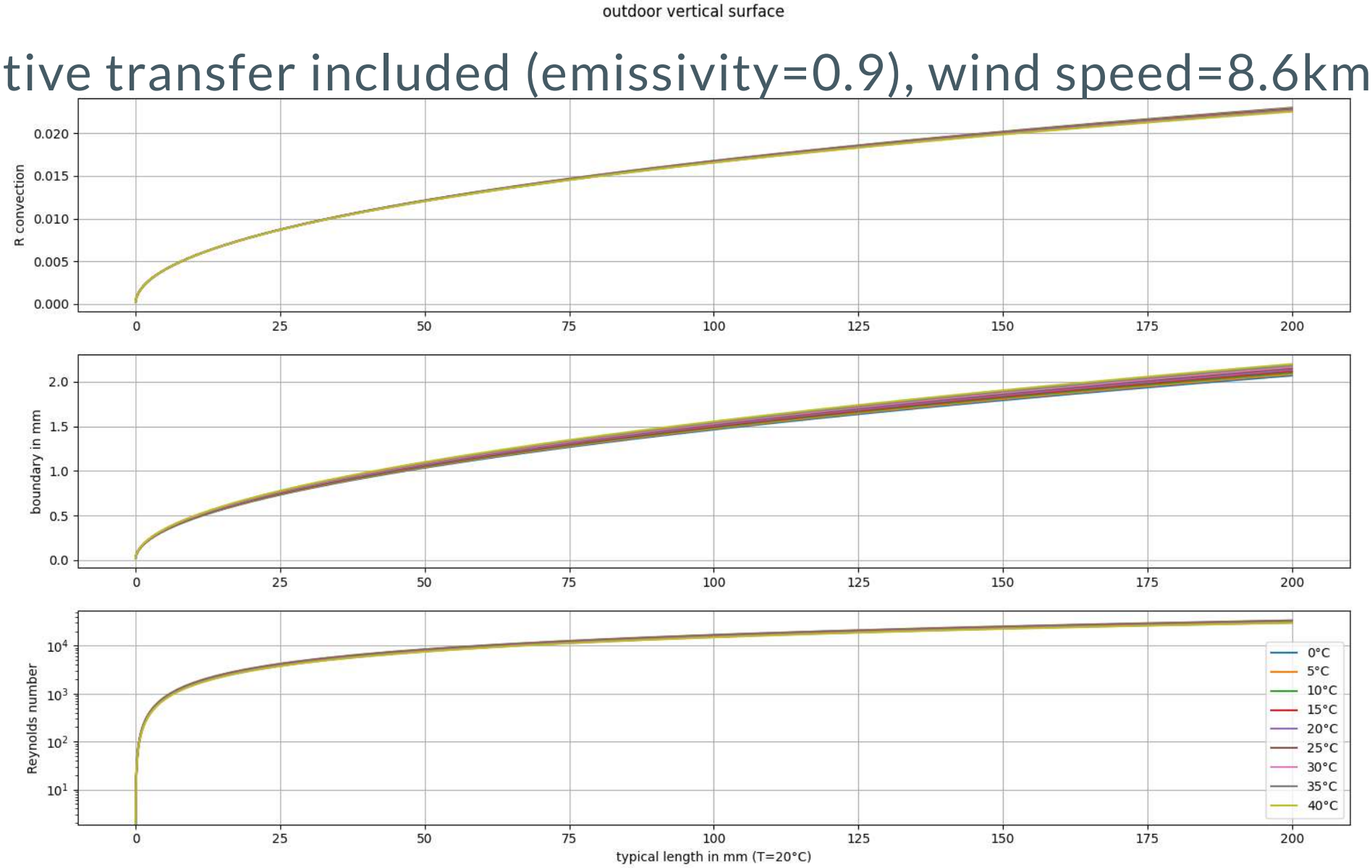
The Reynolds number is given by:

$$R_e = \frac{VL}{\nu}$$

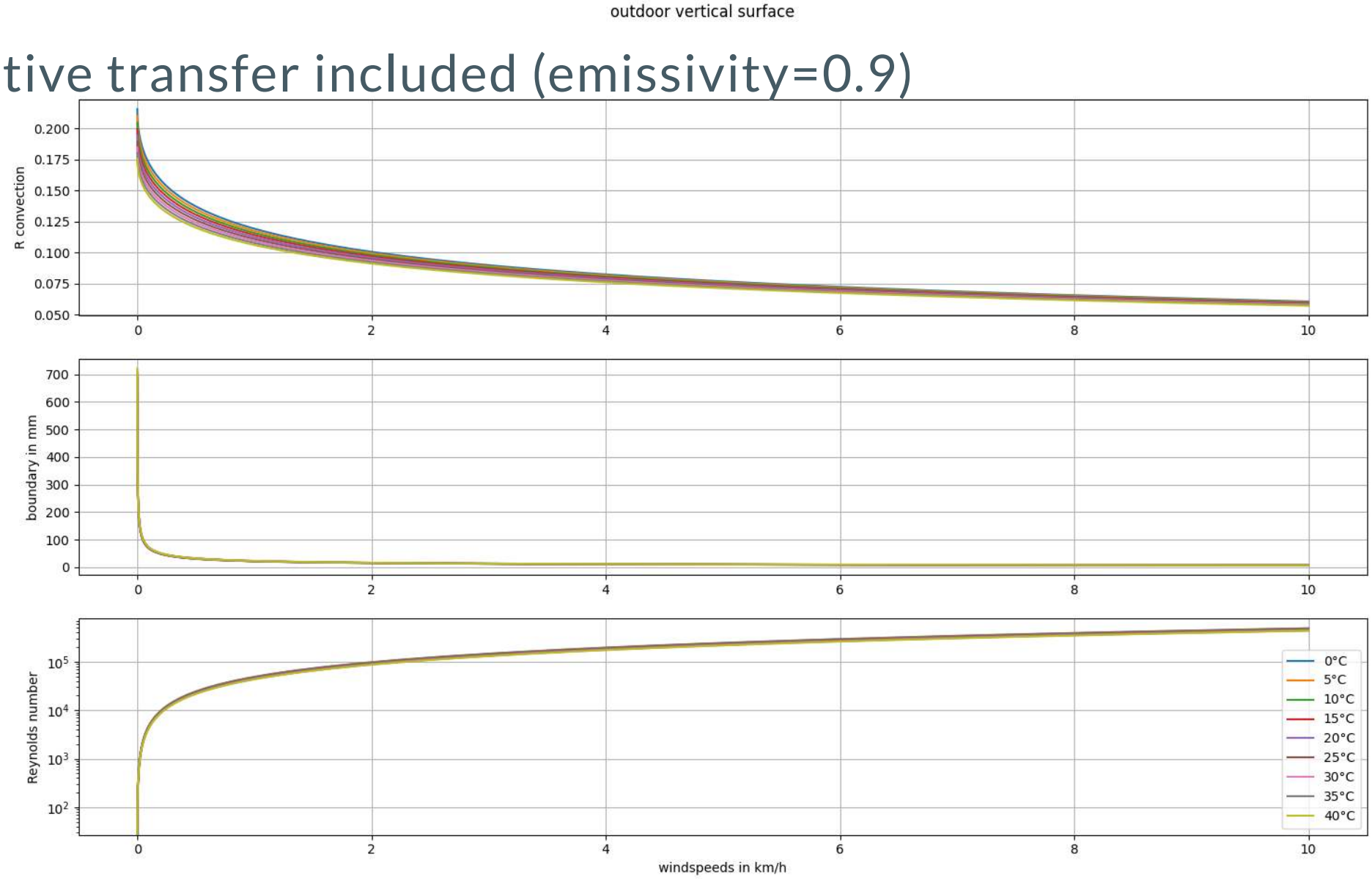
where:

- V is the wind speed
- L is the height of the wall
- ν is the kinematic viscosity of air

Radiative transfer included (emissivity=0.9), wind speed=8.6km/h



Radiative transfer included (emissivity=0.9)



Wind against a horizontal roof

$$R_c = \frac{1}{h_c S}$$

with S is the roof surface, $h_c = \frac{\lambda N_u}{L}$ and $N_u = 0.036 R_e^{0.8} P_r^{0.33}$ ($P_r = 0.71$, the Prantl number)

With radiative transfer, the thermal resistance of the whole boundary air layer:

$$R_L = \frac{1}{(h_c + h_r) S}$$

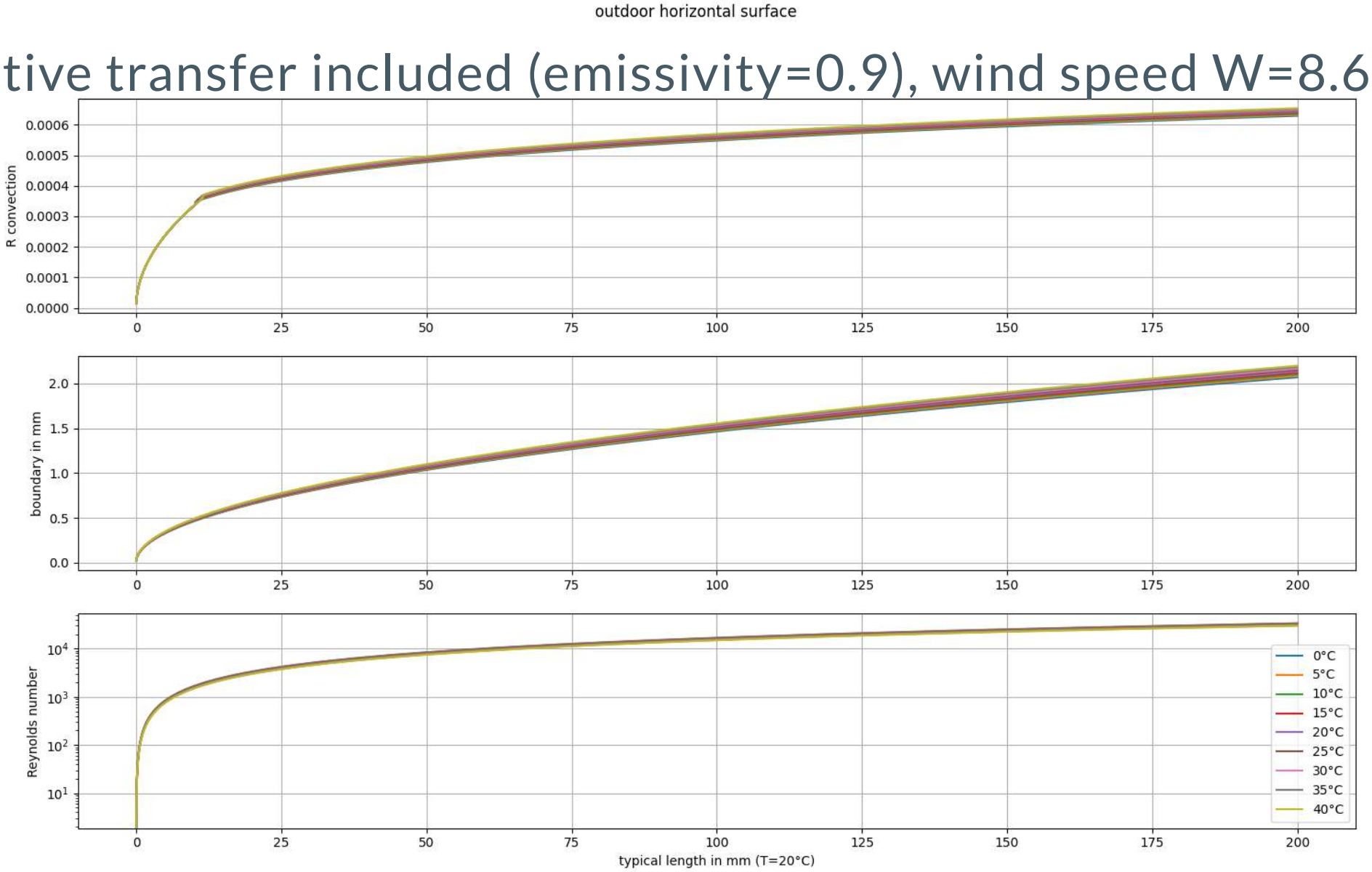
The Reynolds number is given by:

$$R_e = \frac{WL}{\nu}$$

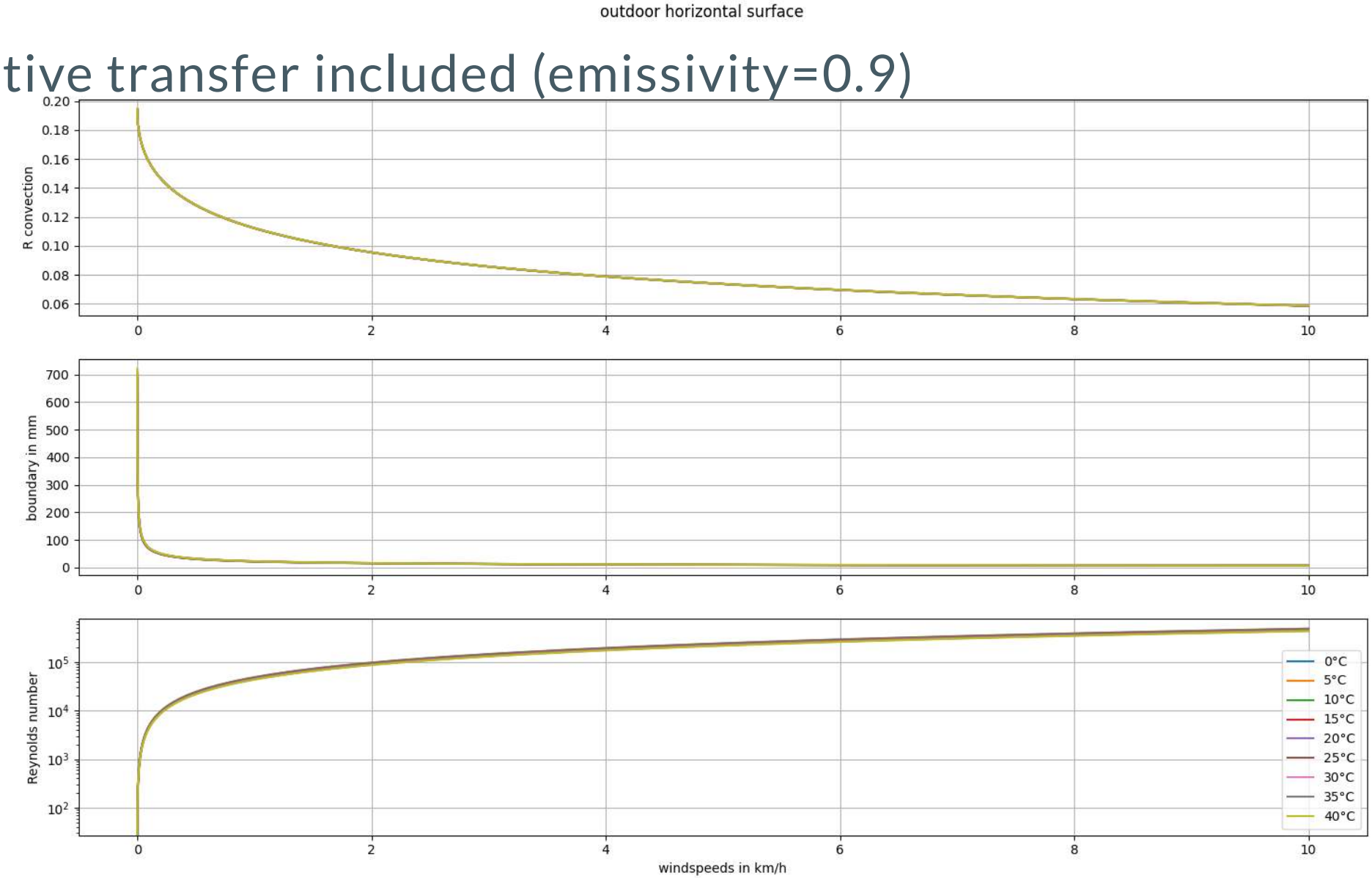
where:

- W is the wind speed
- L is the height of the wall
- ν is the kinematic viscosity of air

Radiative transfer included (emissivity=0.9), wind speed W=8.6km/h



Radiative transfer included (emissivity=0.9)



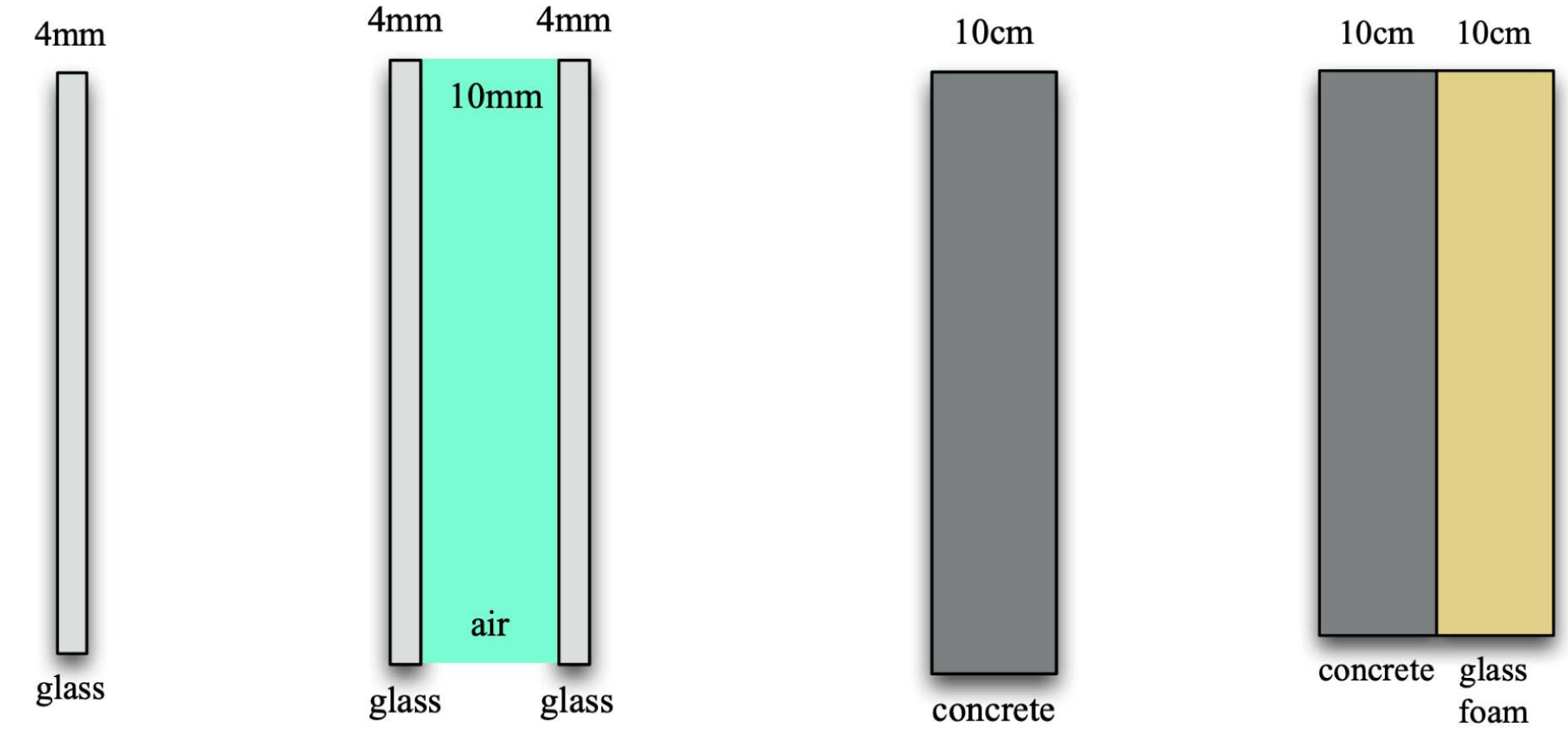
Standard Values to use for RT2012 and RE2020 (emissivities of 0.9 included.)

where	position	h_L	R_L
indoor	vertical	$7.69 W/m^2 K$	$0.13 m^2 K/W$
indoor	horizontal ascending flow	$10 W/m^2 K$	$0.10 m^2 K/W$
indoor	horizontal descending flow	$5.88 W/m^2 K$	$0.17 m^2 K/W$
outdoor	vertical or horizontal	$5.7V + 11.4$ or $25 W/m^2.K$	$1/h_L$ or $0.04 m^2 K/W$

Regarding cavities, if $L \leq 300mm$), according to the ThBat (Th-U 2.2.1.2), an air gap can be assimilated to variable resistance $R = R(L)$:

$L(mm)$	$R(m^2.K/W)$
0	0
5	0.11
7	0.13
10	0.15
15	0.17
$25 \leq L \leq 300$	0.18

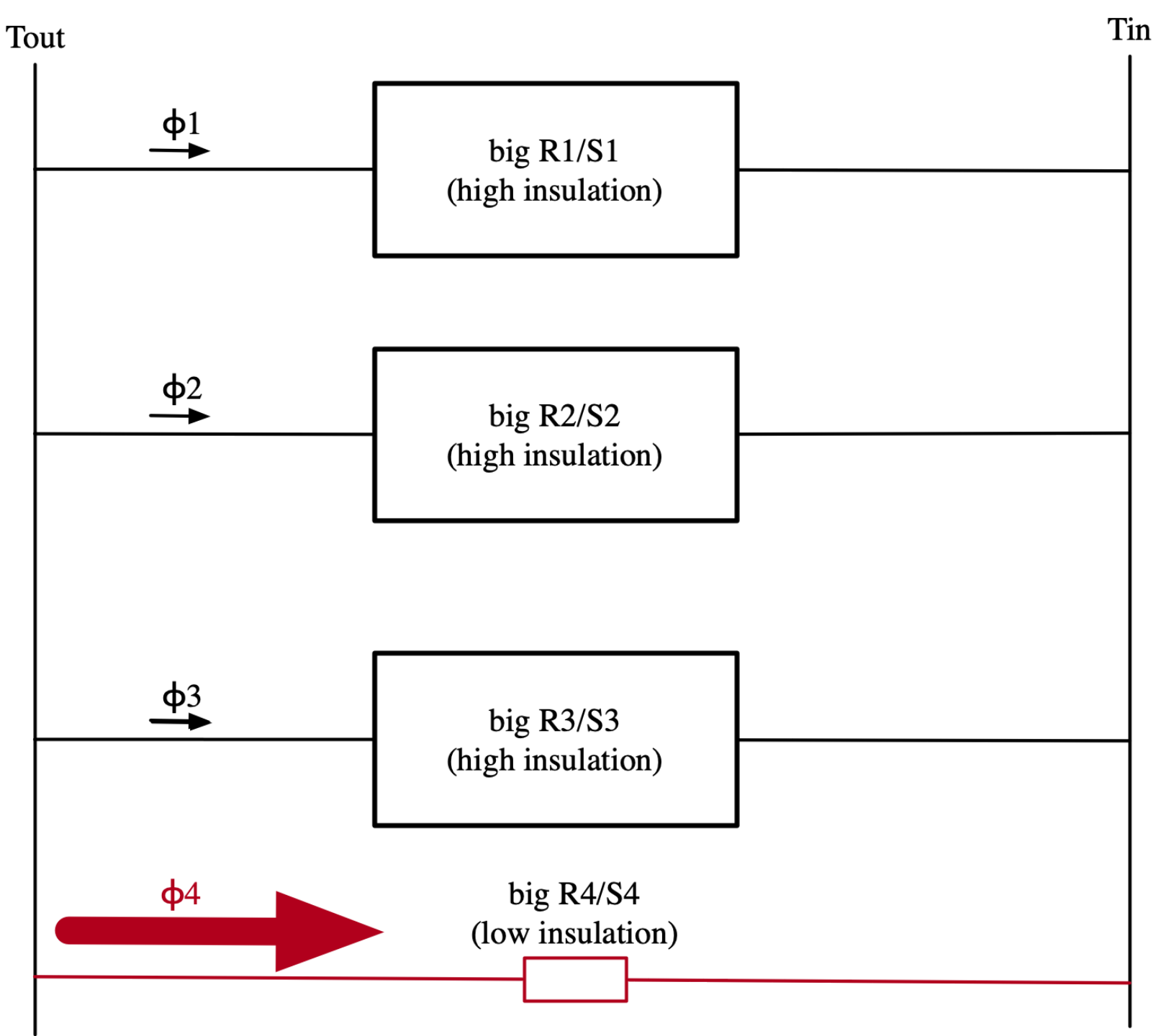
Applying this to double glazing leads to (without considering the boundary air layers):



weak component

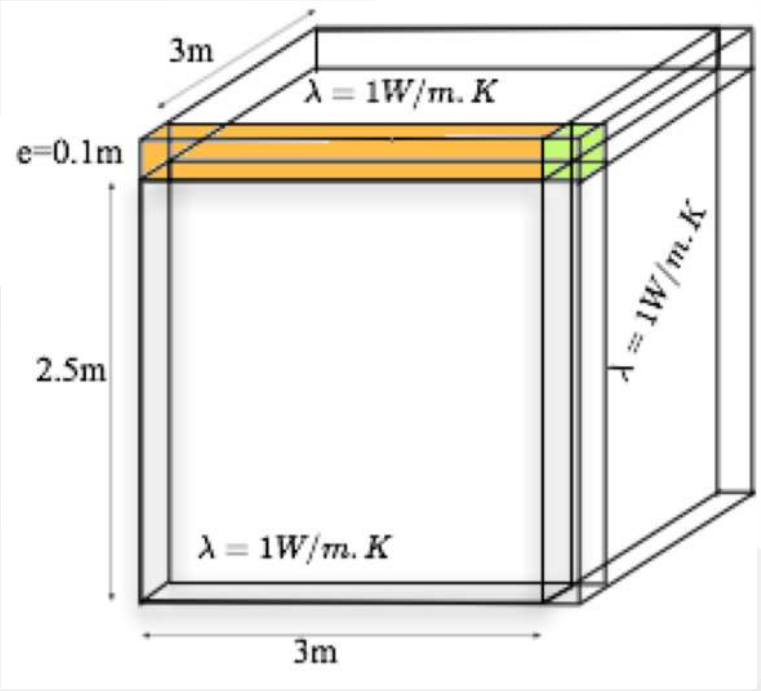
Re-consider the previous configuration but instead of a double glass, a single one is used ($R_g = 0.004 m^2.K/W$ or $U_g = 250 W/m^2 K$ for $1 m^2$). Additionnaly, the insulated wall is reinforced with $R_w = 4 m^2.K/W$ or $U_w = 0.25 W/m^2.K$ for $14 m^2$.

The overall system satisfied:
$US = 250 + 3.5 = 253.5 W/m^2.K \approx 250 W/m^2.K$. Whatever the insulation thickness over the walls, the weak resistance will dominates: during a retrofitting, weak component must be avoided because they determine the thermal losses.


Tout
Tin
ϕ1
big R1/S1
(high insulation)
ϕ2
big R2/S2
(high insulation)
ϕ3
big R3/S3
(high insulation)
ϕ4
big R4/S4
(low insulation)

In case of complex assembly, it's easier:

- to compute the thermal resistance R_i of each layer of a wall
- to sum them up to get the equivalent resistance of each wall component
- to inverse resistance to get transmitivity coefficient U_i
- to sum up the $U_i S_i$ of the components of the same wall (same temperature on both sides)



Walls are easy to model because there is only 1 dimension: the thickness but junctions between walls might have to be modeled in 2D, and possibly in 3D:

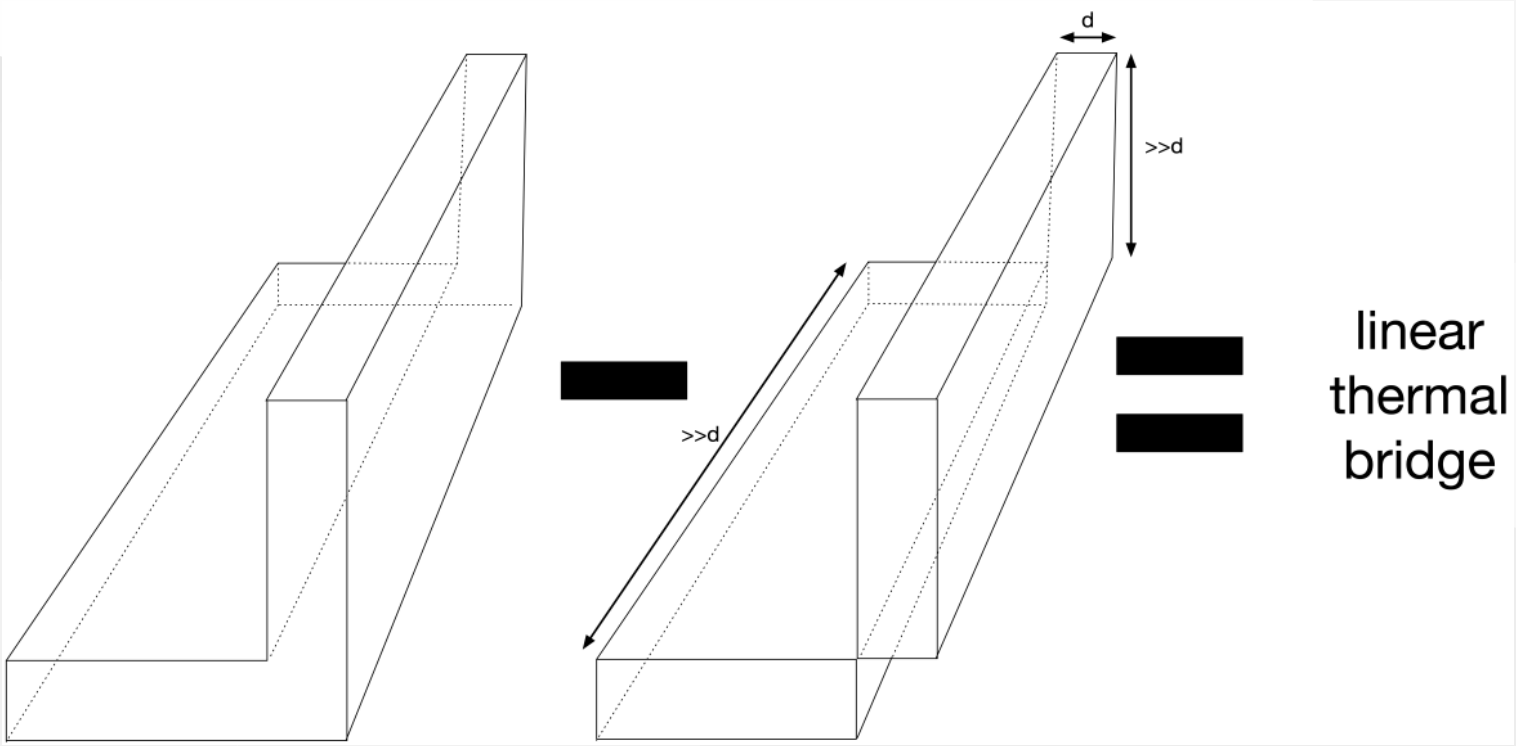
$$\mathrm{div}(\overrightarrow{\mathrm{grad}(\mathrm{T})}) = 0$$

$$\phi = -\lambda \mathrm{grad}(\mathrm{T})$$

The 2D and 3D components are named **thermal bridges**.

2D thermal bridges must be multiplied by length whereas 1D are punctual (and actually not common).

Using finite element method, thermal bridges are computed as:



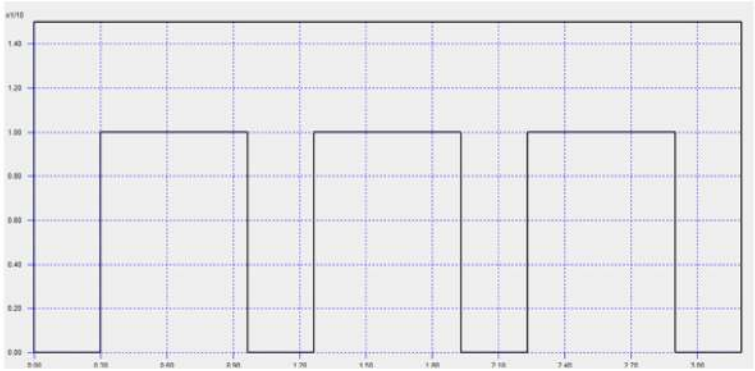
The volume or surface is decomposed into many small elements, that are gathered into big matrices satisfying:
$[M][T] = [N]$.

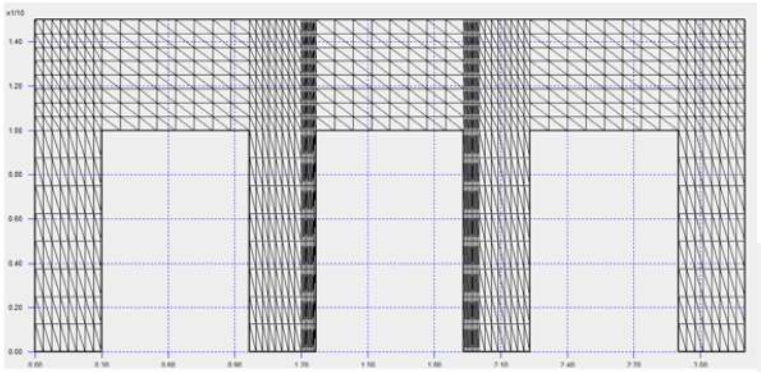
Re-organizing the vector temperatures at nodes like $T' = \begin{bmatrix} T_u \\ T_k \end{bmatrix}$,
where "u" means "unknown temperature" and "k" stands for "known temperatures". We get:

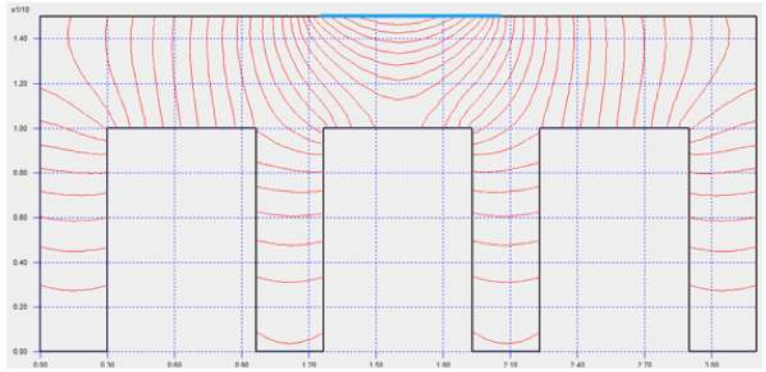
$$\begin{bmatrix} M_{u,u}, M_{u,k} \\ M_{k,u}, M_{k,k} \end{bmatrix} \begin{bmatrix} T_u \\ T_k \end{bmatrix} = \begin{bmatrix} N_u \\ N_k \end{bmatrix}$$

The temperatures at unknown are obtained by solving
$T_u = M_{u,u}^{-1} \left(N_u - M_{u,k} T_k \right)$. Heat flows (in Watt) are obtained with
$\varphi = -\lambda grad(T)$

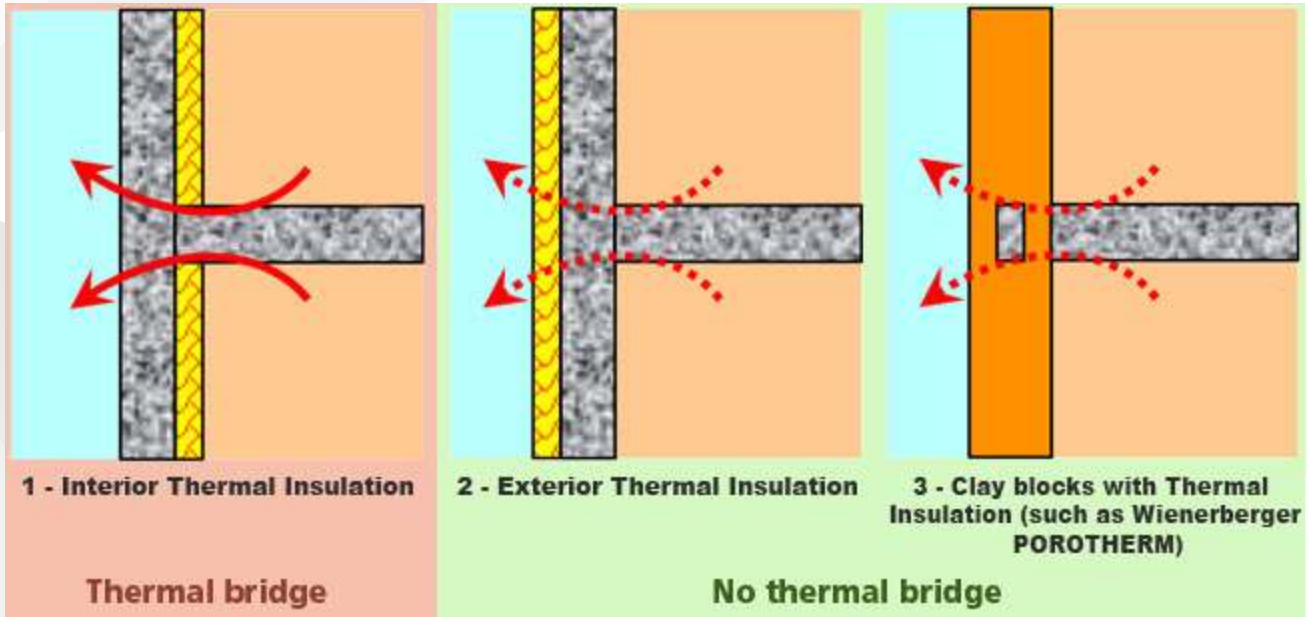
The shape is (1) defined (2) meshed and (3) solved.





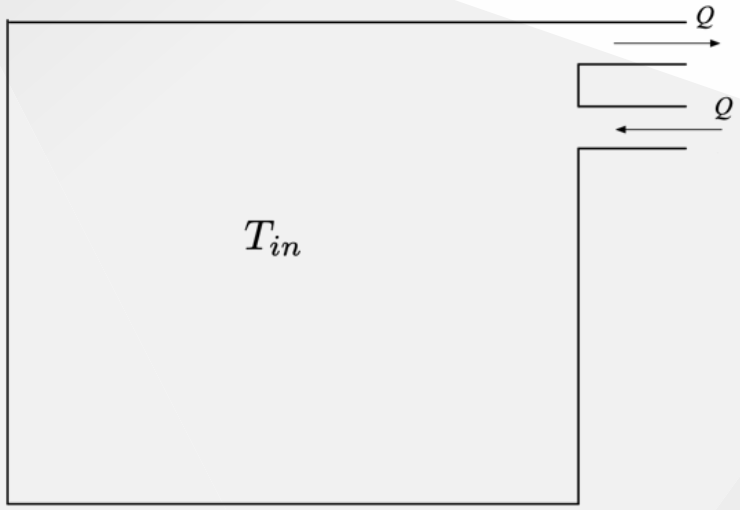


Solutions to avoid the heat leaks through thermal bridges are for instance:

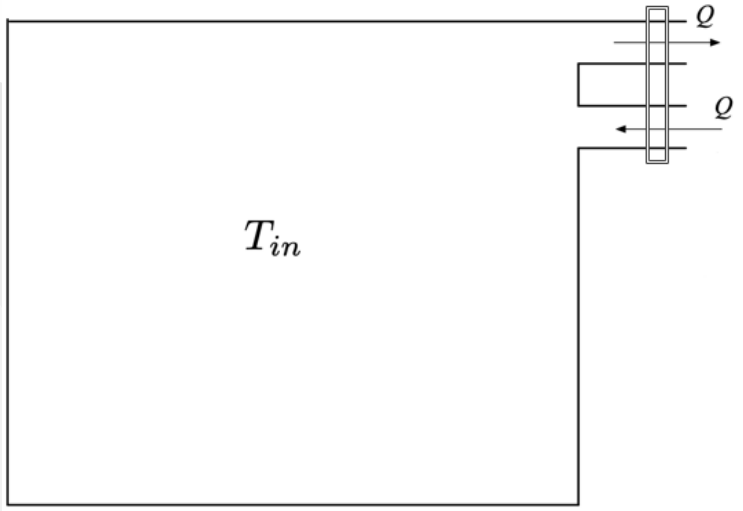


Advection (heat transportation)

Consider a room with a quantity of air $Q(t)$ exchanged from indoor at temperature T_{in} with outdoor at temperature T_{out}. The heat transported by air $\varphi(t)$ is given by: $\varphi(t) = \rho c_p Q(t)(T_{in} - T_{out})$ It corresponds to a thermal resistance $R_v(t) = \frac{1}{\rho c_p Q(t)}$.

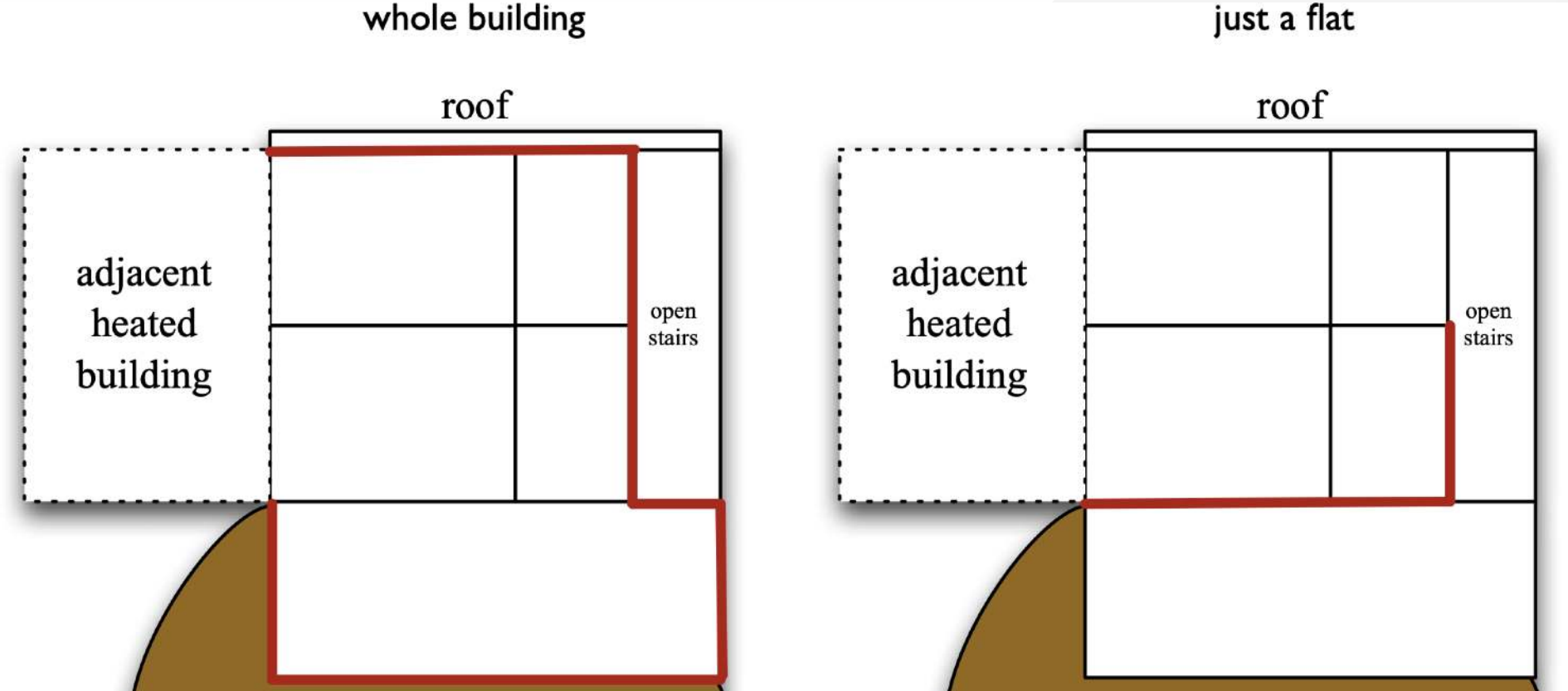


Let's consider there is a heat exchanger that recovers ζ of the heat transported by air. The recovered heat is given by $\zeta\varphi(t)$. Therefore, the thermal resistance of the heat exchanger is $R_v(t) = \frac{1}{(1-\zeta)\rho c_p Q(t)}$.



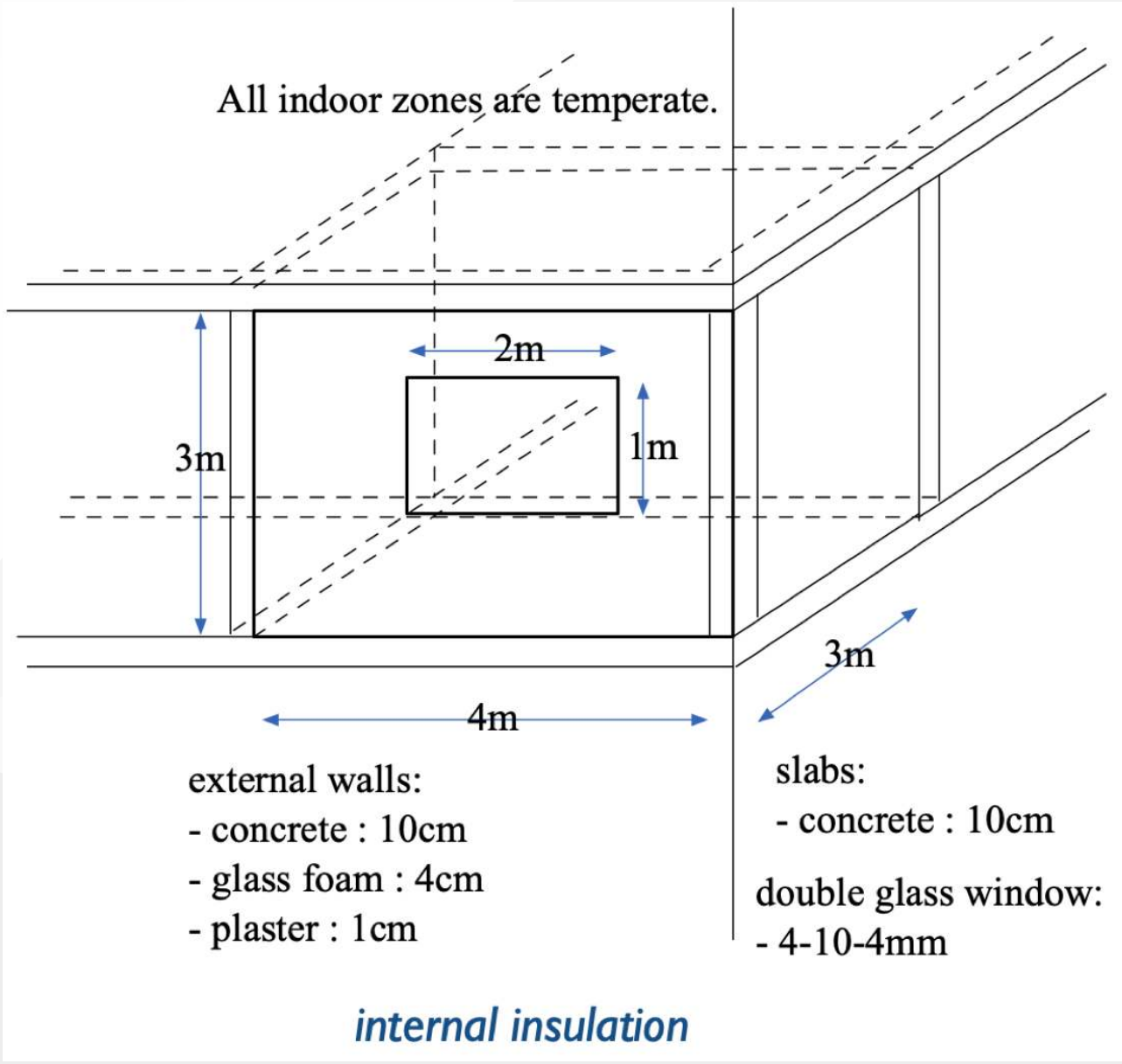
Modeling a whole system

Before starting calculations, the interfaces with heat exchanges have to be identified:



Example:

- determine the interfaces of thermal losses
- compute the transmission coefficient for each surface (neglecting surface phenomena)
- compute the Ubat in W/m2.K

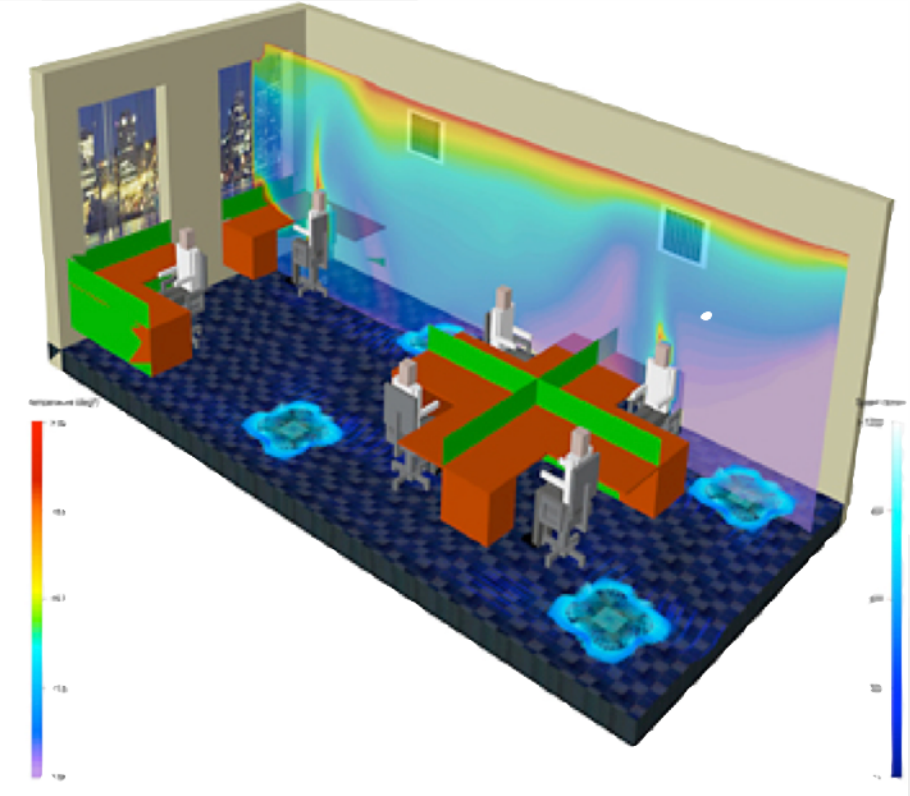


Solution considering only interfaces with thermal losses

- 2 walls without window: 3mx3m+4mx3m-2mx1m
  thickness:0.1m[concrete]+.04m[glass]+0.01m[plaster]
  $R_{wall} = \frac{0.1}{1} + \frac{0.04}{0.05} + \frac{0.01}{0.35} = 0.93K.m^2/W : U_{wall} = 1.08W/m^2.K$
- 1 window 1mx2m,
  thickness:0.004m[glass]+0.01m[air]+0.004m[glass]
  $R_{glazing} = 2\frac{0.004}{1} + 0.15 = 0.158K.m^2/W : U_{glazing} = 6.33W/m^2.K$

$$U_b = \frac{U_{wall}S_{wall} + U_{glazing}S_{glazing}}{S_{wall} + S_{glazing}} = 1W/m^2.K$$

Because reality is too complex and too detailed, simplified modeling is needed.



More for equity, the simplified modeling must be defined by the law.

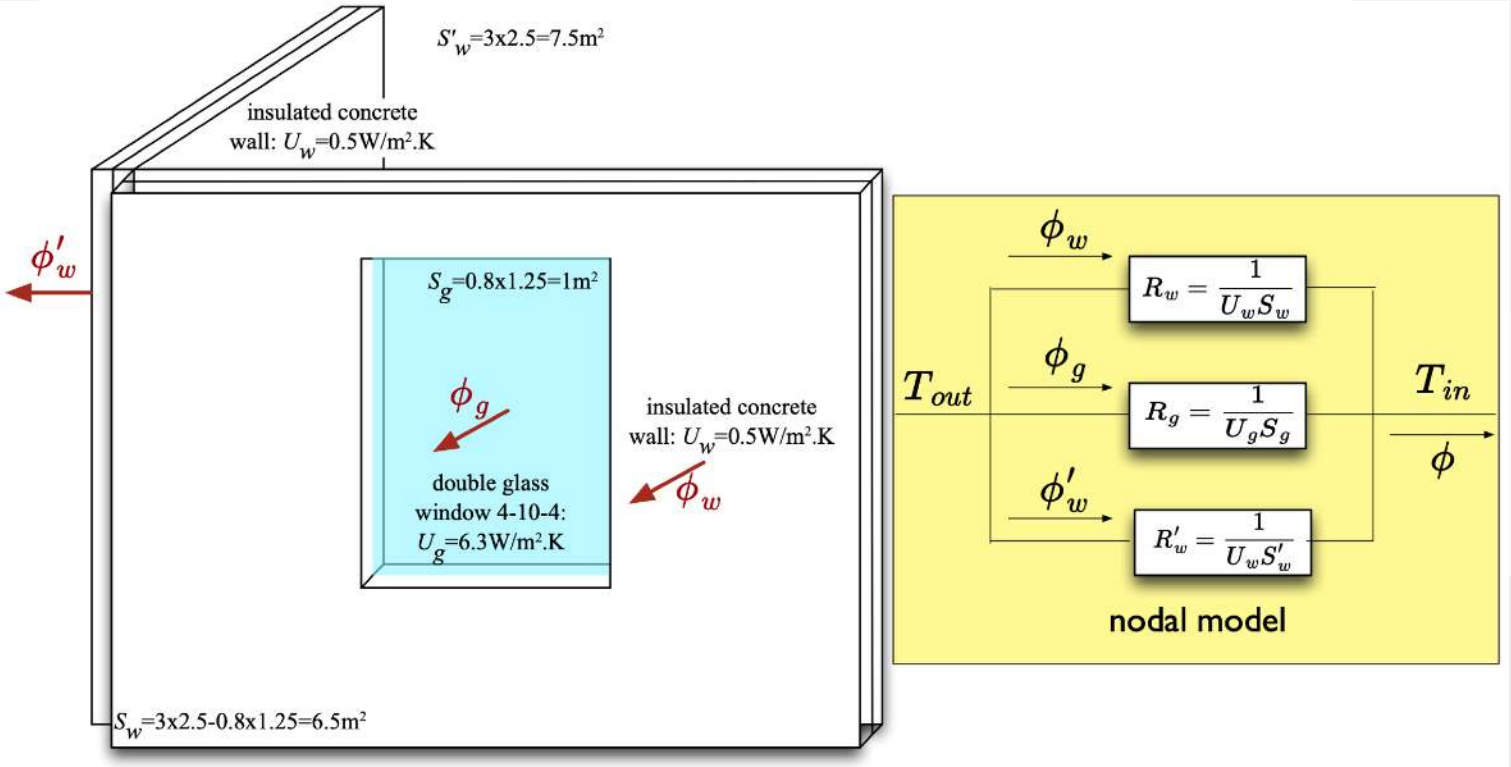
RT2012/RE2020 regulatory calculation method: the ThBAT (CSTB)

- RT2012
- Th-U: Transmission coefficients
  - booklet 1 (p1): Generalities
  - booklet 2 (p25): Materials
  - booklet 3 (p58): Glazed walls
  - booklet 4 (p136): Opaque walls
  - booklet 5 (p240): Thermal bridges

- Th-L: light transmission on walls
- Th-S: solar transmittance of walls
- Th-I: thermal inertia
- Th-BCE: conventional data or thresholds
- RE2020 guide: the guide to RE2020

Calculating complex structures

Let's now consider a multi-component wall:

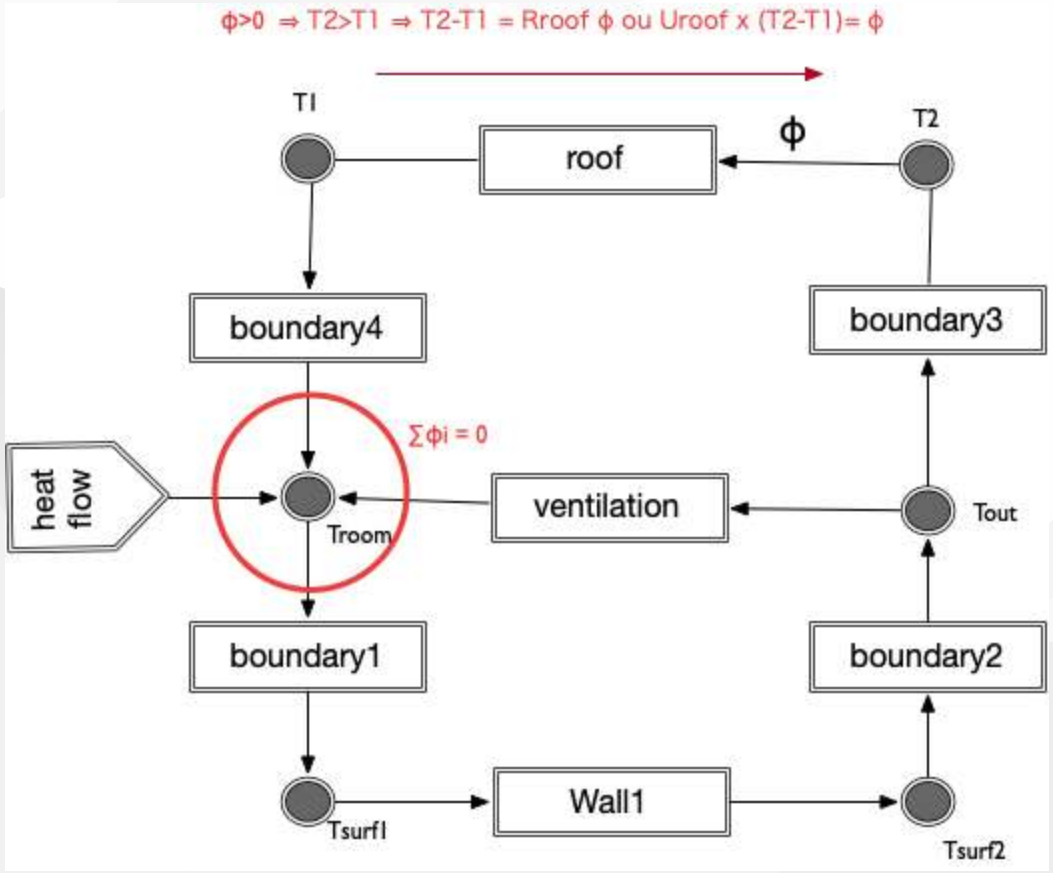


$$\phi = \phi_w + \phi_g + \phi' w = (U_w S_w + U_g S_g + U_w S'_w)\Delta T$$
$$= (3.25 + 6.3 + 3.75)\Delta T = 13.3\Delta T$$

1. Identify the non-adiabatic interfaces (and neglect the other ones)
2. Add the boundary air layers
3. Identify the nodes of temperatures (nodal model): each temperature is related to an unique node
4. Identify the same sequence of interface components by identifying the temperatures on both sides: same temperatures = same interface with different components
5. Identify components of the same interface by identifying heat flow passing through
6. Sum up the thermal resistances (R) of each interface component corresponding to a unique heat flow

7. Invert thermal resistance of interface components to get transmitivity coefficients U
8. Sum up the transmitivity coefficients of components to get a single value per heat flow (i.e. per interface)
9. Add thermal bridges
10. Add air renewal
11. Model your thermal systems with power sources injected at some nodes, place thermal resistance following the "recipient convention":

Thermal and electricity analogy



Going further

RK Rajput. A Textbook of Heat and Mass Transfer [Concise Edition] (p. 443). S Chand & Co Ltd. Kindle Edition.

Self-check questionnaire

12 questions — building heat transfer and modeling

Choose the best answer each time.

(In class: discuss in pairs; answers on the following slides.)

Q1 — Heat transfer modes

According to the opening slides, heat transfer in/build around a shelter includes:

- A. Conduction (in solids), convection (fluids near surfaces), radiation (surfaces / transparent media), plus advection (ventilation/infiltration) and heat storage in materials
- B. Conduction only; convection and radiation are negligible in buildings
- C. Radiation only; walls do not conduct heat
- D. Advection means conduction inside concrete exclusively

Q2 — Conduction vs convection in air

Why is the thermal conductivity λ of immobile air not used directly like for a solid?

- A. Real air layers are not truly immobile: convection dominates near surfaces; insulation traps air in small cavities
- B. Air has no thermal capacity
- C. Fourier's law does not apply to any material
- D. Air conductivity is always higher than copper

Q3 — Fourier’s law

Fourier’s law on the slides relates heat flux density to:

- A. $\vec{\varphi} = -\lambda \,\mathrm{grad}(T)$
- B. $\vec{\varphi} = \lambda T^4$ only
- C. $\vec{\varphi} = h_c(T_{\mathrm{surf}} - T_{\mathrm{air}})$ inside bulk concrete with no gradient
- D. $\vec{\varphi} = \rho c_p Q(T_{\mathrm{in}} - T_{\mathrm{out}})$ in all solids

Q4 — U-value and R-value (single layer)

For a homogeneous layer of thickness e and conductivity λ, the slides give:

- A. $U = \lambda / e$ (W/m²·K) and $R = e / \lambda$ (m²·K/W)
- B. $U = e / \lambda$ and $R = \lambda / e$
- C. U and R are independent of thickness
- D. U is in J/kg·K

Q5 — Multilayer walls

For heat flow in series through several resistances R_i, the equivalent resistance is:

- A. $R_{\text{eq}} = \sum_i R_i$ (then $U = 1/R_{\text{eq}}$ for the assembly)
- B. $R_{\text{eq}} = 1/\sum_i R_i$ always without summing
- C. Resistances multiply: $R_{\text{eq}} = R_1 \times R_2 \times R_3$
- D. Only the thickest layer counts; others are ignored

Q6 — Solar vs terrestrial radiation at surfaces

The slides state that the Sun emits mainly shortwave radiation; building surfaces then:

- A. Absorb shortwave, warm up, and exchange longwave IR with other surfaces and the sky; most glazing is opaque to longwave IR
- B. Reflect all shortwave without temperature change
- C. Emit shortwave to the sky at 288 K
- D. Transmit longwave solar radiation through all opaque walls

Q7 — Linearized surface radiation

The simplified radiative exchange between a surface at T_1 and indoor air at T_{in} is written as:

- A. $\varphi_1 \approx h_{r,1}(T_1 - T_{\mathrm{in}})$ with $h_{r,1} = 4\sigma\varepsilon_1\bar{T}_K^3$
- B. $\varphi_1 = \sigma T_1$ with no temperature difference
- C. $h_{r,1}$ is independent of emissivity
- D. Radiation requires wind speed in the Reynolds number

Q8 — Natural vs forced convection

Which pairing matches the lecture?

- A. Natural convection: buoyancy-driven circulation from temperature/density differences; forced convection: fans, wind, HVAC moving fluid over surfaces
- B. Natural convection requires fans; forced convection requires no fluid motion
- C. Both are identical to conduction in steel
- D. Forced convection occurs only inside solid insulation foam

Q9 — Combined surface coefficient

With convection and radiation at a surface, the boundary layer resistance uses:

- A. $R_L = 1/\big((h_c + h_r)S\big)$
- B. $R_L = h_c + h_r$ without area
- C. $R_L = 1/(h_c S)$ only; radiation is always zero indoors
- D. $R_L = \lambda/e$ for the air film only

Q10 — ThBAT / RT2012–RE2020

ThBAT (CSTB) in this course is presented as:

- A. The regulatory simplified method for thermal calculation used in RT2012 and RE2020 (e.g. Th-U transmission, Th-I inertia…)
- B. A weather file format only
- C. A PMV comfort index
- D. A solar position algorithm unrelated to transmission

Q11 — “Weak component”

The weak component example shows that when glazing has very low R (high U):

- A. It dominates overall losses: improving wall insulation alone may barely change US until the weak element is upgraded
- B. Windows never affect total heat loss
- C. US is always set by the best-insulated wall only
- D. Thermal bridges disappear automatically

Q12 — Advection (ventilation)

Heat transported by ventilation between indoor and outdoor air is modeled on the slides as:

- A. $\varphi(t) = \rho c_p Q(t)\,(T_{\text{in}} - T_{\text{out}})$, equivalent to a resistance $R_v = 1/(\rho c_p Q)$
- B. $\varphi = \lambda\,\mathrm{grad}(T)$ in the air duct only
- C. $\varphi = \sigma T^4$ for ventilation rates
- D. Independent of volumetric flow rate Q

Answer key (1/2)

Q	Answer	Short rationale
1	A	Conduction, convection, radiation, advection, storage.
2	A	Air: convection + trapped air in insulation; not bulk λ alone.
3	A	Fourier: flux proportional to $-\mathrm{grad}\, T$.
4	A	$U = \lambda/e$, $R = e/\lambda$.
5	A	Series resistances add.
6	A	SW absorption → LW exchange; glazing opaque to LWIR.

Answer key (2/2)

Q	Answer	Short rationale
7	A	Linearized $h_r(T_1 - T_{\text{in}})$, $h_r = 4\sigma\varepsilon\bar{T}^3$.
8	A	Buoyancy vs forced flow (wind, fans).
9	A	$R_L = 1/((h_c + h_r)S)$.
10	A	ThBAT = CSTB regulatory thermal method (RT2012/RE2020).
11	A	High-U glazing can dominate US.
12	A	Ventilation enthalpy flux $\rho c_p Q\,\Delta T$.